COMPUTATIONAL ELECTROMAGNETIC MODELING
FOR WIRELESS CHANNEL CHARACTERIZATION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
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Graduate School of The Ohio State University

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* * * * *

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ABSTRACT

A new full wave methodology and a well-established ray-tracing method are employed for indoor wireless communications channel modeling. The full-wave method, referred to as array decomposition-fast multipole method (AD-FMM) for indoor simulation, is based on the finite element-boundary integral formulation. A key feature of this technique is the use of domain decomposition methods to efficiently model repeatable components such as bricks, chairs, tables, etc. This leads to significant memory reduction allowing the simulation of realistic structures with different antenna locations to predict the statistical profiles of the received signal strength. These profiles are subsequently used to evaluate the bit error rate (BER) for specific digital modulation schemes. The method is also employed to predict the statistical channel capacity for multiple input multiple output (MIMO) systems via the complementary cumulative distribution function. This dissertation also exploits an established ray-tracing electromagnetic (EM) simulation tool, and measurements for indoor channel characterization for wireless applications. Specifically, measurements are conducted for indoor environments to validate the channel model obtained using ray tracing tools. Such ray-tracing channel models are appropriate for 4th generation 60 GHz communication systems.
This is dedicated to my family and those who have inspired me.
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FIELDS OF STUDY

Major Field: Electrical Engineering

Studies in:
- Electromagnetics
- Communications
- Applied Statistics
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CHAPTER 1

INTRODUCTION

1.1 Motivation and Challenges

Increasing demand for high-speed wireless communication systems and the advent of multiple input multiple output (MIMO) systems imply strong interest for more accurate predictions of channel propagation in the multipath-rich indoor environment. A typical indoor environment is depicted in Fig. 1.1. As illustrated, interactions between penetrable structures such as furniture, walls, etc. may occur, resulting in multipath and fading. These effects can seriously degrade communication system performance operating inside the environment. One can do little to eliminate multipath interference, but careful characterization of the channel can be used to mitigate undesirable effects.

This dissertation is focused on predicting channel characterization with particular emphasis on using rigorous numerical methods. These characterizations are then used to predict bit error rates (BERs) in MIMO set-ups.
1.2 Brief Review of Related Work

So far, characterization of indoor propagation channels has been mostly carried out using measurements [2–39] since numerical modeling has by and large been restricted to simple configurations (see for example [40–47]). A few research groups [40–43] have proposed using ray-tracing methods to study indoor propagation and some others have considered two-dimensional finite difference time domain (FDTD) methods [44–47] to evaluate field distributions within wireless communication environments. The calculated data are subsequently processed to extract a variety of statistics useful for site planning.

As can be understood, measurements are tedious and provide a limited dataset for statistical extraction. Alternatively, simulations can allow for data collections
under many different site configurations by easily changing the source location and local geometry. Therefore, they hold promise for extracting more representative statistical parameters. Among the available analysis methods, ray tracing techniques are quite popular for urban microcellular environments [48–58] due to their computational speed. However, ray tracing methods are rather approximate for complex indoor environments and are not suitable for inhomogeneous dielectric structures such as furniture and reinforced walls. Rigorous full wave methods such as the 2-D FDTD [44–47] have been pursued instead for these situations. However, their CPU requirements are quite high and simplifications may be necessary for carrying out the analysis.

1.3 Dissertation Contribution

In this dissertation, we propose a full wave methodology for indoor channel modeling, and the application of a well-established ray-tracing code (i.e. numerical electromagnetic code-basic scattering code (NEC-BSC)). The full-wave method, referred to as array decomposition-fast multipole method (AD-FMM) [59,60] is based on the finite element-boundary integral (FE-BI) formulation [61]. AD-FMM incorporates several features, practical for modeling large complex structures with repeatable components. Specifically, AD-FMM can buildup complex structures (walls, furniture, etc.) constructed from a set of repeatable components (chairs, bricks forming a wall, etc.). By storing only the repeatable components and not the entire structure (see Fig. 1.2), significant memory reduction is achieved. As such, it allows for full-wave analysis of large computational domains with practical geometrical features and within reasonable CPU time using FMM and its variants (via the application of the Fast Fourier
Transform (FFT) for handling the matrix vector products associated with the overall FE-BI matrix). Consequently, typical communication channel parameters can be extracted from simulations and herein, we demonstrate an application of the AD-FMM for a realistic classroom configuration.

Using the AD-FMM dataset, we can extract channel characteristics such as the power delay profile (PDP), and parameters such as the mean excess delay and root mean square (RMS) delay spread. These convey the fading conditions of the channel which affect the bit error rate (BER), or the information loss in a communication system. We present the average BER estimation for different modulation schemes and antenna locations within the classroom configuration. The channel statistics caused by multipath (statistically modeled by Rayleigh, Ricean, or Nakagami distributions) can be extracted directly from the calculated data using the power delay profiles (PDPs). We also extract important statistics regarding the multipath effect, occurring within the environment to extract the cumulative distribution function of the signal strength. Both the PDP and the fading channel model are employed when considering candidates for the digital communication modulation/demodulation schemes. To illustrate use of the extracted statistics, we consider various binary modulated signals over different fading channels and present the average BERs for each fading channel. Overall, it is shown that the extracted statistical parameters are affected not only by obstacles obstructing the propagation path, but also on the antenna location, and especially the antenna height.

The multipath (occurring within an indoor environment) statistics are also affected by the signal bandwidth [62]. This is particularly relevant to the possible 60 GHz wireless band, considered for 4th generation (4G) systems. The bandwidth
at this frequency is around 7 GHz, implying a greater need for accurate channel characterization. Like most other communication systems, channel characterization at these millimeter (mm) wave frequencies has primarily been done using measurements [63–67] since rigorous numerical analysis methods are prohibitive at these frequencies. Here, we employ a well-established ray-tracing (RT) method for channel modeling at these frequencies. We also compare extracted channel parameters based on the RT model with newly collected measurements, and other measurements from [63]. We then proceed to present pertinent fading statistics for typical indoor channels (i.e. single room and hallway) at 60 GHz.

1.4 Dissertation Outline

The objectives of this dissertation are to (1) understand multipath effects within a site-specific indoor environment; (2) create an accurate propagation model for high
speed wireless communication systems; (3) evaluate the wireless communication system performance via bit error rate (BER) calculation based on the propagation model; (4) recommend approaches to improve communication links. With these goals in mind, we have successfully developed a systematic approach as addressed in this dissertation.

Chapter 2 presents a new full wave methodology for indoor channel modeling. This method, referred to as array decomposition-fast multipole method (AD-FMM) is based on the finite element-boundary integral (FE-BI) formulation. The latter can efficiently model repeatable components, allowing for practical modeling of in-building objects such as walls, furniture, people, etc. As such, a specific indoor environment containing these objects is chosen to demonstrate the AD-FMM capability. Based on this representative example, we also extract time dispersive channel parameters and channel statistics, resulting from multipath effects. These parameters and statistics are important pertaining to the optimal design of wireless communication systems.

To illustrate the use of extracted statistics, we consider various binary modulated signals over different fading channels, and present the average BER for each fading channel. This allows one to gauge the performance of the wireless systems via channel statistics. Various ways for improving the channel throughput, such as the placement of transmitting antennas and deployment of more transmitting antennas, are investigated as well.

Chapter 3 explores the use of AD-FMM to evaluate the channel statistics for multiple input multiple output (MIMO) systems. The channel statistics are extracted via the Kolmogorov-Smirnov (KS) goodness-of-fit test. To illustrate the use of channel statistics in MIMO settings, we compute the MIMO channel capacity via Monte-Carlo
simulations for various configurations. Specifically, the spacings between antennas (i.e. transmitter or receivers) are investigated in this chapter.

Chapter 4 proposes the use of a ray-tracing (RT) method to study indoor propagation at the millimeter frequencies. Based on the collected data, we extract channel parameters and channel statistics. In this chapter, we also compare channel parameters based on the RT model with in-house collected measurements. Two indoor propagation channels are considered, namely that of an empty room and that of hallway.

Chapter 5 gives a summary of the presented work along with conclusions and recommendations for future work.
In this chapter, we propose a new full wave methodology for indoor channel modeling. The method, referred to as array decomposition-fast multipole method (AD-FMM) [59, 60] is based on the finite element-boundary integral (FE-BI) formulation [61]. The latter incorporates several features that are practical for modeling structures with repeatable components. Specifically, AD-FMM can model structures (walls, furniture, etc.) constructed from a set of repeatable components (chairs, bricks forming a wall, etc.). By storing only the repeatable components and not the entire structure (see Fig. 2.1), significant memory reduction is achieved. As such, it allows for full-wave analysis of large computational domains with practical geometrical features and within a reasonable CPU time using FMM and its variants (via the application of the Fast Fourier Transform (FFT) for handling the matrix vector products associated with the overall FE-BI matrix). Thus, the typical channel parameter extraction is possible and this is demonstrated in this chapter with a specific example.

In the following section, we briefly introduce the AD-FMM for indoor channel analysis, and proceed to carry out a full wave characterization of a typical classroom.
that includes (1) an array of chairs and (2) models for the dielectric walls. Based on this representative example, we can extract channel parameters such as mean excess delay and root mean square (RMS) delay spread (as later shown in Section 2.4). These parameters convey the fading conditions of the channel which can degrade the quality of the signal, causing loss of capacity in a communication system. As a result, there are ways to measure the capacity of the system (i.e. system performance), and among others is the bit error rate (BER). Thus, Section 2.6 gives the average BER estimation (via channel statistics) for different modulation schemes and the antenna locations within the classroom configuration. This channel variability is caused by the multipath (statistically modeled by Rayleigh, Ricean, or Nakagami distributions) and can be extracted directly from the power delay profiles (PDPs). Here, we employ these important statistical data (which describe the multipath behavior) occurring within the environment to extract the cumulative distribution functions. The latter can also reveal information pertaining to signal loss and the statistical behavior of the propagation channel. Both the signal loss estimation and the fading channel model are employed when considering candidates for digital communication modulation/demodulation schemes. To illustrate the use of the extracted statistics, we consider various binary modulated signals over different fading channels and present the average BERs for each fading channel. Overall, it is shown that the statistical parameters are affected not only by obstacles obstructing the propagation path, but also can depend significantly on the transmitting antenna height.

So far, all these studies described above are pertaining to the analysis of a single transmitting antenna. As is well-known, one can improve the reliability of the system by an additional deployment of transmitting antennas. Therefore, Section 2.7
investigates the use of multiple transmitting antennas in the wireless communication systems via the aforementioned approach (i.e. the full-wave EM analysis via AD-FMM → channel statistics extraction → BER calculation). Section 2.8 concludes this chapter.

In the following section, we propose a rigorous full-wave method to compute electric and magnetic fields for different site-specific configurations.

### 2.1 Introduction to AD-FMM

As is well-known, modeling of dielectric (possibly inhomogeneous) structures requires full-wave methods which are computationally intensive and impractical for large scale complex configurations (as is the case for indoor channels). Recently, a method referred to as array decomposition method (ADM), was introduced for full-wave electromagnetic (EM) modeling containing very large repeatable structures such as finite antenna arrays [59]. The key concept of ADM is to store only the single array element description which can then be repeated in the computational process without a need to regenerate matrix elements. Furthermore, if the array elements are uniformly separated (constant distance between the unique array elements), the resulting system matrix for the interactions within the array will have a block-Toeplitz form. This can be exploited (via the FFT) for significant central processing unit (CPU) time reductions. In this case, a room could be modeled as arrays of ceiling, floor tiles, wall panels, chairs, and human subjects. The interactions among the various components forming the scattering configuration, will depend on the type of arrangement being considered, the number of different unique elements (bricks, chairs, human models,
Antenna located at (x=5m, y=5m, z=2m)

(a) Chair repeatability with domain decomposition.

Wall section repeated to form the entire wall.

Use of domain decomposition to exploit the BI block Toeplitz matrix property

(b) Illustration of wall modeled using individual bricks of the same type.

Figure 2.1: Two examples of domain decomposition for modeling indoor environments.
etc.) being used, and the material composition of the individual elements, just to mention a few.

Mathematically, the matrix structure for the field analysis of an array of chairs or wall sections (as displayed in Fig. 2.1) takes the form

\[
\begin{bmatrix}
  [a]_{11} & [a]_{12} & \cdots & [a]_{1M'} \\
  [a]_{21} & [a]_{22} & \cdots & [a]_{2M'} \\
  \vdots & \vdots & \ddots & \vdots \\
  [a]_{M1'} & [a]_{M2'} & \cdots & [a]_{MM'}
\end{bmatrix}
\begin{bmatrix}
  \{e\}_1 \\
  \{e\}_2 \\
  \vdots \\
  \{e\}_M
\end{bmatrix}
= 
\begin{bmatrix}
  \{b\}_1 \\
  \{b\}_2 \\
  \vdots \\
  \{b\}_M
\end{bmatrix},
\]

where the submatrices \([a]_{pq}\) represent interactions between the repeatable blocks or elements forming the structure, \(\{e\}_p\) is the column of unknown field coefficients within the repeatable block, and \(\{b\}_p\) is the excitation vector due to the incident field from the antenna(s) or some other radiators. As usual, \([a]_{pq}\) refers to the self-coupling submatrices, whereas the submatrices \([a]_{pq|p\neq q}\) describe the interactions among the \(p^{th}\) and \(q^{th}\) blocks. If all the blocks are identical and equally spaced, then \([a]_{pq}\) would only depend on the distance between the blocks viz. on \(p - q\). As a result, within the matrix in (2.1), only the 1\(^{st}\) row and 1\(^{st}\) column are needed to uniquely describe the matrix. Moreover, the matrix is identified as block-Toeplitz. Consequently, standard FFT-like structures can be adapted [59, 60] to rapidly carry out the matrix-vector products. As such, one can carry out the analysis of extremely large structures with \(O(N)\) memory and \(O(N \log N)\) CPU requirements.

Any analysis method, including the method of moments (MoM) [68] or the finite element method (FEM) or its hybrid version finite-element boundary-integral (FE-BI) method [61], can be used for the characterization of the repeatable blocks. Given our experience with the hybrid finite element-boundary integral (FE-BI) method, we adapted this approach to make use of existing computer codes. In accordance with
the FE-BI method, the volumetric region is modeled via the FEM, with the radiation condition enforced by the introduced boundary integral and tested on the boundary of the volumetric block. As such, the individual submatrices in (2.1) take the form

\[
[a]_{pq|p=q} = \begin{bmatrix}
A_{EE}^{EE} & A_{EE}^{ES} & 0 \\
A_{SV}^{EE} & A_{SS}^{EE} & A_{SS}^{EH} \\
0 & A_{SS}^{HE} & A_{SS}^{HH}
\end{bmatrix},
\] (2.2)

\[
[a]_{pq|p\neq q} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & A_{SS}^{HE} & A_{SS}^{HH}
\end{bmatrix}.
\] (2.3)

In these matrices, the superscripts $EE$ and $EH$ imply electric-to-electric and electric-to-magnetic field interactions, respectively. Referring to [69], $A_{VV}^{EE}$ describes the interactions among the fields within the volume. Also, $A_{VS}^{EE} = (A_{SV}^{EE})^T$ refers to the interactions between the volume and surface elements of the block and $A_{SS}^{EE}$ gives the surface-to-surface or boundary-to-boundary interactions. The matrices $A_{SS}^{EH,HE,HH}$ are fully populated and are responsible for the rigorous nature of the FE-BI method, but are concurrently responsible for the larger computational requirements. The reader is referred to [61,69], and [70] for further details. The aforementioned ADM approach can be further accelerated with the incorporation of the fast multipole method (FMM). The FMM integration was considered in [60] and this approach allows for faster evaluations of the array interactions. The resulting method is referred to as array decomposition-fast multipole method (AD-FMM) and has been extensively tested for radiation by finite arrays. We remark that the matrix in (2.1) can, in general, be applied to include several periodic sections, each with a different unique block [59,60], and this is exploited in this work. In case of realistic situations where movable objects (such as chairs and tables) may lead to irregularities, AD-FMM is
not expected to break down. Chairs and/or tables can be moved around in any arbitrary orientation, but the memory to store the geometry of the furniture remains the same. However, the CPU time will increase since the Toeplitz property may no longer hold. The interested readers are referred to [59,60,71] for more details on the computational aspect of the AD-FMM. Next, we proceed to validate AD-FMM for indoor propagation modeling.

2.2 Validation of AD-FMM

As a first step, we proceed to validate the AD-FMM using the standard FE-BI method for structures related to its proposed application. For this validation, an infinitesimal dipole antenna was placed at (0.2,0.2,0.5) and the total fields were calculated at 2.4 GHz. In this context, we consider the wall (shown in Fig. 2.2) that

Figure 2.2: Here is the validation setup. On the right are the 3D model via FE-BI (on the top) method and AD-FMM (at the bottom). For AD-FMM, the wall is decomposed into 4 equal smaller sub-walls in this example.
is sufficiently small for the analysis using standard FE-BI method. Refer to Fig. 2.2, the wall has a height of 0.8m and a length of 1.05m with 0.145m thickness. The material of the wall is characterized by an $\epsilon_r = 4.0$ and $\sigma = 2.0\text{S/m}$. As can be realized, we compare the difference between the solutions via (2.4), where $\mathbf{E}^F$ and $\mathbf{E}^A$ denote the total fields obtained based on FE-BI method and AD-FMM, respectively. Here, the fields used for comparison are the sum of all electric field components, viz. $|E_x| + |E_y| + |E_z|$.

$$\frac{||\mathbf{E}^F - \mathbf{E}^A||_2}{||\mathbf{E}^F||_2} = \sqrt{\sum ((E_x^F - E_x^A)^2 + (E_y^F - E_y^A)^2 + (E_z^F - E_z^A)^2)} / \sqrt{\sum ((E_x^F)^2 + (E_y^F)^2 + (E_z^F)^2)}.$$  \hspace{1cm} (2.4)

For both methods, an iterative solver (i.e. bi-conjugate gradient (Bi-CG)) was employed to generate a solution within a specified margin of error. In our calculations, the convergence of Bi-CG was achieved, and the residual error was given by:

$$\frac{||\{b\} - [A]\{x\}||_2}{||\{b\}||_2} < 0.01,$$  \hspace{1cm} (2.5)

where $[A]$ is the system matrix, $\{b\}$ is the forcing vector and $\{x\}$ being the solution estimate. Using this relation, we determine the total fields (measured along a line 1m from the wall) at the presence of this wall (see Fig. 2.2) via the standard FE-BI method and AD-FMM. Employing (2.4), we found that the resulting difference between the solutions of FE-BI method and AD-FMM is less than 1% (see Fig. 2.3).

Next, we compare the storage requirement for both FE-BI method and AD-FMM. Both methods were ran on the Itanium dual-900MHz processors PC clusters at the Ohio super-computer center (OSC). The results based on the standard FE-BI method,
Figure 2.3: Comparison of total fields between both FE-BI and AD-FMM methods.
utilized 6 GBytes of shared memory, whereas the AD-FMM used 0.5 Gbytes. That is 10 times memory saving.

Next, we examine the matrix fill time for both FE-BI method and AD-FMM. In this example, based on the speed of Itanium machines, the FE-BI method required 40 mins to fill the matrix, while AD-FMM method took merely 4 mins for the same operation, implying a 10-fold CPU reduction as well.

In this example, the entire simulation time for the FE-BI method and the AD-FMM were 7.8 hrs and 6 mins, respectively.

With such reductions, we can subsequently proceed to handle a larger indoor propagation problem, as demonstrated in the next section.

2.3 Modeling Approach for the Classroom

We proceed now with the full-wave analysis of a classroom based on the same approach. The two examples of interest are (1) an empty room and (2) the same room occupied with 20 chairs. The dimensions of the room are depicted in Fig. 2.4 with the detailed floorplan displayed in Fig. 2.5. The room is of length 9.0m, width 4.5m and height 2.1m. The walls, floor, and ceiling are 7cm thick characterized by an $\epsilon_r = 4.0$ and $\sigma = 0.01\text{S/m}$. Each of the 9.0m $\times$ 2.1m walls has 5 windows 0.9m $\times$ 0.7m in size. To model the room using AD-FMM, 3 types of repeatable blocks were used (to construct the walls). We will refer to these as Type I, II and III. Type I was of size of 0.9m $\times$ 0.7m $\times$ 0.07m, Type II was of size of 0.7m $\times$ 0.9m $\times$ 0.07m and Type III was of size of 0.9m $\times$ 0.9m $\times$ 0.07m. Using these blocks, 50 Type I, 30 Type II and 100 Type III blocks were required to model the walls and the ceiling/floor. In doing so, 304,000 unknowns, requiring a total of 5.53 Gbytes in memory, were used for
modeling of the entire classroom. The chairs were modeled as a single (Type IV) block and then repeated as necessary to form the rows and columns. For the propagation analysis, an antenna was placed at one corner of the room \((0.0, 0.0, 0.8)\), transmitting a CW Gaussian pulse of bandwidth 500 MHz at 2.4 GHz. As can be realized, a total of 63 different receiving locations were used as depicted in Fig. 2.6 (simulated PDPs are depicted in Fig. 2.7). At each location, we chose the peak envelope of the PDP to be used later in constructing the statistical parameters. Two simulation were conducted – one for the empty room and another for the same room with 20 chairs. In both case studies, the simulation times for the empty and furnished rooms were 0.68 hrs and 2.14 hrs, respectively.
Figure 2.5: Layout of the room used to form the propagation channel. The room is occupied with 20 equally spaced chairs (see Fig. 2.4 for a 3D view).

At each of these locations, the \((E,H)\) field vectors were calculated for three heights: \(z=0\)m, 1m and 2m.

Figure 2.6: Map of the locations where calculations were recorded (see Fig. 2.7).
Figure 2.7: Scatter plot of received power versus time (power delay profile) for the occupied room shown in Fig. 2.4 with the transmitter located at (0,0,0.8).
2.4 Channel Parameters for the Classroom

Multipath channel parameters such as mean excess delay and RMS delay spread are useful in describing the overall characteristics of the multipath profile and are essential in developing design guidelines for digital wireless communication systems. The mean excess delay is the first moment (center of mass) of the power delay profile (PDP), defined here as

\[
\bar{\tau} = \frac{\sum_{k=1}^{M} \tau_k PDP(\tau_k)}{\sum_{k=1}^{M} PDP(\tau_k)} - \tau_A,
\]

(2.6)

where \(\tau_M\) is the last sample along the PDP axis which has significant amplitude and \(\tau_1\) is the first point after the first detectable signal which arrives at time \(\tau_A\).

The RMS delay spread is the square root of the second moment of the PDP which is also the standard deviation, defined as

\[
\sigma = \left[ \frac{\sum_{k=1}^{M} (\tau_k - \tau_{PDP})^2 PDP(\tau_k)}{\sum_{k=1}^{M} PDP(\tau_k)} \right]^{\frac{1}{2}}.
\]

(2.7)

\(\sigma\) is a measure of the delay spread in the propagation channel, and is used to design against inter-symbol interference. Typical values of the RMS delay spread are on the order of microseconds in the outdoor propagation channel [72, 73] and on the order of nanoseconds in the indoor propagation channel [74–77].

These channel parameters can be extracted directly from the PDP. The PDP (in time domain) is equal to \(\frac{1}{2}(\bar{E}^2 + \mu|\bar{H}|^2)\), where \((\bar{E}, \bar{H})\) are the total electric and magnetic field vectors, respectively. In the following section, we propose a rigorous full-wave method to compute the frequency domain electric and magnetic fields for different site-specific configurations. We then proceed with the computation of the aforementioned PDPs.
2.5 Fading Statistics for the Classroom Due to Multipath Effects

As is well-known, indoor propagation environments can be characterized by scattering from irregular features, such as furniture, multiple reflections, and interactions among objects. As a result, signals arrive at the receiver through several paths, causing constructive or destructive interference, or channel fading. It has been shown by many authors that the distribution of the received signal power obeys the Rayleigh distribution [78, 79] when there is no line of sight (LOS) between the transmitter and receiver, or a very large amount of multipath. If there is a LOS path, the distribution is better described by a Ricean probability density function (PDF). Our key objectives are to examine the features of these using the proposed rigorous analysis method and to compare the statistics of our simulations to the Rayleigh and Ricean distributions [78, 79].

The PDF of a Ricean distribution is

\[
p(r) = \begin{cases} \frac{r}{\sigma^2} \exp \left[ -\frac{r^2 + A^2}{2\sigma^2} \right] I_0 \left( \frac{Ar}{\sigma^2} \right) & ; \quad A \geq 0, r \geq 0 \\ 0 & ; \quad r < 0 \end{cases},
\]

(2.8)

where \( r \) is the received amplitude signal with the mean power of the multipath signal related to the variance of the distribution. The constant, \( A \), denotes the peak amplitude of the dominant signal, and \( I_0(\bullet) \) is the modified Bessel function of the first kind and zero order. An alternative way to describe the Ricean distribution is by using the \( K \)-factor (\( K = \frac{A^2}{2\sigma^2} \)). When \( K = 0 \), it is well-known that the Ricean distribution reduces to the Rayleigh distribution as shown

\[
p(r) = \begin{cases} \frac{r}{\sigma^2} \exp \left( -\frac{r^2}{2\sigma^2} \right) & ; \quad 0 \leq r \leq \infty \\ 0 & ; \quad r < 0 \end{cases}.
\]

(2.9)
We next proceed to compare our extracted statistics with the Rayleigh and Ricean distributions. To do so, a cumulative distribution function (CDF) was built based on the received power density. As is already known [78, 79] from narrowband measurements, indoor propagation is best described using Rayleigh distribution (when there are many multipath and none of it is dominant), or Ricean distribution (when there is a dominant LOS). Thus, in proceeding to describe the PDF of the received signals using our full-wave model, we will employ the Chi-square ($\chi^2$) fitness test to these distributions.

In the Chi-square test, the relative degree of fitness is defined by

$$\chi^2 = \sum_{j=1}^{k} \frac{(O_j - E_j)^2}{E_j},$$

(2.10)

where $O_j$ is the value of the CDF as depicted by the bar height, $E_j$ is the best fitted value, and $k$ is the last sample along the CDF. Clearly, the value of $\chi^2 = 0$ corresponds to a perfect fit. On the other hand, when $\chi^2$ becomes larger, this corresponds to an increasing disparity among the actual and fitted values.

Using the Chi-square goodness-of-fit test, we found that the standard derivation ($\sigma$) of the Rayleigh distribution (see Fig. 2.8) was 23.55 (with $\chi^2 = 5.26$) and 23.6272 (with $\chi^2 = 4.75$) for the empty and occupied rooms, respectively, and the transmitter located at (0, 0, 0.8). Similarly, when we fit the empirical results on the Ricean distribution (see Fig. 2.8), we found that the $K$ factors were 11.78 (with $\chi^2 = 0.1564$) and 11.67 (with $\chi^2 = 0.1239$) for the empty and occupied rooms, respectively. Clearly, the Ricean distribution approximated the fading statistics better for the indoor environment of interest. As is well-known, the Rayleigh distribution is a special case of the Ricean distribution. Therefore, in subsequent studies of the fading conditions
Figure 2.8: Cumulative distribution function (CDF) computed from received power densities in Fig. 2.7. The depicted solid lines come from the least square curve-fit. The darker line represents the Ricean distribution and the light one denotes the Rayleigh distribution.
due to antenna heights (refer to Fig. 2.4), we fit the empirical results only to the Ricean distribution.

To study the fading conditions of the propagation channels for antenna heights, we conducted an extensive simulation campaign (the obtained Ricean parameters and $\chi^2$ are tabulated in Table 2.1). From this series of simulations, it is interesting to note that the $K$-factor (which conveys the fading conditions) range is nearly the same for both the empty and occupied (i.e. with 20 chairs) rooms. Therefore, one can conclude that the chairs do not obstruct the propagation path in those transmitting locations since the $K$-factors do not change noticeably.

To show where the chairs may have an effect, we conducted another set of simulations. Specifically, the receiver location was placed at $(8.0, 4.0, 0.6)$. For those simulations, we recorded $K = 2.74$ and $K = 0$ for empty and occupied rooms, respectively. That is the chairs do obstruct the propagation paths, giving rise to more multipath. As a result, the Ricean distribution tends to approach that of Rayleigh distribution (i.e. $K = 0$), implying that there is no dominant signal. This observation was also noted in the measured data given in [80].

Another observation drawn from our statistical analysis was that the fading conditions were unique at different transmitting heights. For example, the maximum $K$-factor was not found at the highest (or lowest) transmitting location. This is because the ceiling (or the floor) also contributes to multiple reflections. In fact, transmitting at $(0, 0, 0.9)$ and $(0, 0, 0.6)$ caused the most significant components for the empty and occupied rooms, respectively (as also seen from high $K$-factors, implying that there is a higher dominant signal). As a result of our investigation, we can conclude that
Table 2.1: Fading Statistics of empty and occupied rooms as shown in Fig. 2.4. The occupied room is of the same dimension, but with additional 20 chairs.

<table>
<thead>
<tr>
<th>Tx location</th>
<th>Empty room</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma$</td>
<td>$A$</td>
</tr>
<tr>
<td>(0,0,0.3)</td>
<td></td>
<td>5.76</td>
<td>21.83</td>
</tr>
<tr>
<td>(0,0,0.5)</td>
<td></td>
<td>5.23</td>
<td>24.06</td>
</tr>
<tr>
<td>(0,0,0.6)</td>
<td></td>
<td>5.40</td>
<td>26.18</td>
</tr>
<tr>
<td>(0,0,0.7)</td>
<td></td>
<td>6.06</td>
<td>27.47</td>
</tr>
<tr>
<td>(0,0,0.8)</td>
<td></td>
<td>6.14</td>
<td>29.80</td>
</tr>
<tr>
<td>(0,0,0.9)</td>
<td></td>
<td>5.90</td>
<td>30.02</td>
</tr>
<tr>
<td>(0,0,1.5)</td>
<td></td>
<td>6.24</td>
<td>24.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tx location</th>
<th>Occupied room</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma$</td>
<td>$A$</td>
</tr>
<tr>
<td>(0,0,0.3)</td>
<td></td>
<td>5.48</td>
<td>22.10</td>
</tr>
<tr>
<td>(0,0,0.5)</td>
<td></td>
<td>4.73</td>
<td>24.13</td>
</tr>
<tr>
<td>(0,0,0.6)</td>
<td></td>
<td>5.12</td>
<td>26.15</td>
</tr>
<tr>
<td>(0,0,0.7)</td>
<td></td>
<td>6.00</td>
<td>27.54</td>
</tr>
<tr>
<td>(0,0,0.8)</td>
<td></td>
<td>6.10</td>
<td>29.47</td>
</tr>
<tr>
<td>(0,0,0.9)</td>
<td></td>
<td>6.11</td>
<td>29.92</td>
</tr>
<tr>
<td>(0,0,1.5)</td>
<td></td>
<td>6.14</td>
<td>24.87</td>
</tr>
</tbody>
</table>

an optimum transmitting height for different configurations (propagation channels) exists, an observation which can be used for site-planning.

Next, we proceed with the application of the derived statistical model to extract bit error rates.
2.6 Bit Error Rate Calculation for Single Antenna System

An important parameter in any digital communication system is the bit error rate (BER). Using the extracted $K$-factor, we are in a position to extract BER for specific coding systems and for the given indoor environment we just analyzed. To do so, we relied on available work given in [81, Ch.5], [82]. In this section, we consider the modulation scheme based on the binary frequency shift keying (BFSK) and the binary phase shift keying (BPSK) over the chosen fading channels (see Appendix B). The analysis given in [81, Ch.5], [82] takes advantage of the Laplace transforms and/or Gauss-Hermite quadrature integrals evaluations. Based on the aforementioned approach, BERs over fading channels modeled by Rayleigh and Nakagami-$(m, n, q)$ distributions in the general form can be readily obtained from [81, Ch.5], [82]. As is known in [83], the Nakagami-$n$ is a more general representation of a Ricean distribution. However, for the analysis here, we use a more simple integral expression for the BER where the $K$-factor (of the Ricean distribution) is explicitly present. To do so, in the Appendix B, we derive an alternative BER expression that is easier to use. We should point out though, that the BER analysis is not limited to binary signals, but can also be extended to $M$-ary signals. The reader is referred to [81, Ch.5] for BER calculations for $M$-ary signals. As shown in Appendix B, the BER for a Ricean fading channel is given by

$$P_2 = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \phi}{\sin^2 \phi + g^2} \right) \exp \left[ -Kg \sin^2 \phi \right] d\phi. \quad (2.11)$$

Fig. 2.9 presents the BER for Ricean fading channels with $K = 8$, 10, and 12 (refer to (B.13)) and gives the calculated BER versus signal-to-noise ratio per bit.
Figure 2.9: Bit error rate (BER) calculation for Ricean fading channels with $K = 8, 10, 12$. We observe that as $K$ increased from 8 to 12, BER is reduced by a factor of 100. We also note that locating the transmitting antenna at $z = 0.3$ versus $z = 0.9$ (implying a change in $K$ from 7.17 to 12.94) leads to a BER reduction by a factor of 100 for the empty room. For the occupied room, the reader is referred to Table 2.1. Based on the work presented here, our data indicates that through simulation, we can determine the optimal transmitting location to achieve the highest possible data rate across the site-specific propagation channel in a wireless digital communication system.
2.7 Bit Error Rate Calculation for Multiple Antennas System (MAS)

In this section, we employ the AD-FMM to characterize multiple antenna systems in a complex indoor channel. To model the multipath within these systems, we adopt a multi-Ricean fading approach (will be discussed later in Section 2.7.2). Specifically, we consider various binary modulated signals over the multi-Ricean fading channel. In doing so, the average bit error rate (BER) can also be computed. We also examine specifically the dependency of the capacity reliability on the number of antennas used as a function of transmitting locations.

2.7.1 Classroom Description with MAS

In this study, of particular interest was the correlation of furniture (inside the classroom) with fading conditions. Thus, we considered two configurations: (1) an empty room and (2) the same room occupied with 20 chairs. The dimensions of the room are depicted in Fig. 2.4 and the detailed floorplan (with antenna locations depicted) is displayed in Fig. 2.10. The room had a length of 9.0m, width 4.5m and height 2.1m. The walls, floor, and ceiling were 7cm thick and characterized by a relative dielectric constant ($\epsilon_r$) of 4.0 and conductivity 0.01S/m [1]. Each of the 9.0m × 2.1m walls had 5 windows 0.9m × 0.7m in size. For the proposed propagation analysis, four antennas were placed at the corners of the room, transmitting continuous wave (CW) Gaussian pulses of bandwidth 500 MHz at 2.4 GHz operating frequency. A typical power delay profile (PDP) is depicted in Fig. 2.12. A total of 81 such PDPs were recorded at different receiving locations (for statistical modeling) as depicted in Fig. 2.11. At each location, we chose the peak value of the PDP to
be used later for the implementation of the maximum likelihood estimation (MLE) of the statistical parameters. All simulations to compute the PDPs were carried out on an Itanium dual-900MHz PC cluster at the Ohio Super-computer Center (OSC). For reference, the total number of unknowns for these simulations were 276, 800 and 304, 100 for the empty and furnished rooms, respectively and the central processing unit (CPU) times for the empty and furnished rooms were 2.72 hrs and 8.56 hrs, respectively.

### 2.7.2 Fading Statistics of MAS for the Classroom

As can be understood, indoor propagation is governed by the interactions among furniture, walls or other objects, thereby resulting in multipath. Because of these multipath, signals arrive at the receiver with different phase, or time delay, causing
At each of these locations, the \((E, H)\) field vectors were calculated for three heights: \(z=0\text{m}, 1\text{m} \) and \(2\text{m}\).

Ant.1-(0.0,0.0,1.5)
Ant.2-(0.0,4.0,1.5)
Ant.3-(8.5,0.0,1.5)
Ant.4-(8.5,4.0,1.5)

Figure 2.11: (a) Map of the locations where calculations were recorded; The first receiver location at the bottom row is at \((0,0,0,0,0)\), and each location is 100cm apart; For the middle receiver locations, the first is at \((2.25,0,0,0,0)\) with the rest also placed 100cm apart; Finally, for the top receiver locations, the first is at \((4.0,0,0,0,0)\) with the rest placed 100cm apart.
Figure 2.12: Power delay profile (PDP) for the occupied room shown in Fig. 2.4 with the transmitter located at (0, 0, 1.5) and the receiver at (4.0, 2.25, 1.0).
fading. This fading can be obtained from the PDPs by first developing a histogram based on the probability of the received energies above a pre-determined threshold level (see Fig. 2.8). Next, we look for the best-fit of the observed histogram (by means of MLE) to a Ricean distribution. Ricean distribution was chosen because it represents the statistics of this study much better than the Rayleigh distribution (as shown in Fig. 2.13 and Fig. 2.14). To check how well the observed and estimated Ricean data fit, we performed a null hypothesis testing, \( H_0 : (\text{Observed data} = \text{fitted Ricean}) \) vs. the alternative hypothesis \( H_A : (\text{Observed data} \neq \text{fitted Ricean}) \). The chi-square \( (\chi^2) \) goodness-of-fit test was subsequently used for this hypothesis testing. We found that the data and estimated Ricean distributions were within the 1% significant level of the \( \chi^2 \) goodness-of-fit test. As such, we proceeded to use the extracted statistics to mitigate the fading effects.

To combat fading conditions, we explored several antennas for transmission and reception. For our analysis, four antennas were placed at each corner of the room as depicted in Fig. 2.6 (namely, Antenna 1 at \((0.0, 0.0, 1.5)\), Antenna 2 at \((0.0, 4.0, 1.5)\), Antenna 3 at \((8.5, 0.0, 1.5)\), and Antenna 4 at \((8.5, 4.0, 1.5)\)). The antennas were represented by an infinitesimal dipole with vertical polarization as the source. As such, no cross-coupling between the antennas was expected. Thus, we could model the multipath in the indoor environment with 4 propagation channels (each associated with different antennas). Bearing this in mind, we conducted an extensive simulation campaign for the classroom channel with (or without) chairs, resulting in the Ricean distributions tabulated in Table 2.2. We remark that as described in Section 2.7.1, the parameters, \( \sigma, A \) and \( K \) were extracted from the received signal collected at the
Figure 2.13: Cumulative distribution function (CDF) computed from peak received PDPs in Fig. 2.12. The depicted solid line comes from the least square curve-fit to Rayleigh CDF.
Figure 2.14: Cumulative distribution function (CDF) computed from peak received PDPs in Fig. 2.12. The depicted solid line comes from the least square curve-fit to Ricean CDF.
81 locations indicated in Fig. 2.6 due to the transmitting antenna given to the left of Table 2.2.

From Table 2.2, it is interesting to note that the $K$-factors (which convey the fading conditions) remain nearly unchanged in the empty and occupied rooms for Antennas 1-4, implying that the chairs do not obstruct the signal paths significantly for these antenna locations. However, the $K$-factor is significantly influenced by the chairs when the receiving device is located behind them (i.e. at $(8.0, 4.0, 0.6)$). In this particular case, we recorded $K = 2.74$ and $K = 0$ for the empty and occupied rooms, respectively. That is, the chairs do obstruct the propagation paths and as a result, weaken the dominant signal. As such, the Ricean distribution tends to approach that of Rayleigh (i.e. $K \to 0$), implying no dominant signal. This observation was also noted in the measured data given in [80].

Another observation is that the fading conditions are unique for each transmitting antenna position. We remark that the maximum $K$-factor corresponds to the antenna located at $(0.0, 0.0, 1.5)$, whereas the minimum $K$-factor corresponds to its location at $(0.0, 4.0, 1.5)$. That is, the dominant signal is attenuated when transmitting at $(0.0, 4.0, 1.5)$ since the $K$-factor is lower.

The above extracted $K$-factors were computed from the peak PDP values. On the other hand, if we have used all PDP values (i.e. for the case of occupied room), the resulting cumulative distribution function (CDF) is shown in Fig. 2.15, with a comparison to the least square fitted to a Ricean CDF. From Fig. 2.15, the resulting $K = 7.96$ as compared to the value of $K = 8.12$ extracted from the peak PDP. Nevertheless, the difference in BERs corresponding to either of these values is insignificant. Thus, either of these approaches in computing the BER is applicable in this study.
$\sigma = 7.24$

$A = 28.89$

$K = 7.96$

Figure 2.15: Cumulative distribution function (CDF) based on all PDP values. Also shown (bold) is the CDF based on the least square curve fit to Ricean CDF.
Table 2.2: Fading Statistics of the empty and occupied rooms as shown in Fig. 2.4. The occupied room is of the same dimension, but with additional 20 chairs.

<table>
<thead>
<tr>
<th>Tx location</th>
<th>σ</th>
<th>A</th>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty room</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ant.1-(0.0,0.0,1.5)</td>
<td>6.24</td>
<td>24.79</td>
<td>7.90</td>
</tr>
<tr>
<td>Ant.2-(0.0,4.0,1.5)</td>
<td>6.30</td>
<td>12.16</td>
<td>1.86</td>
</tr>
<tr>
<td>Ant.3-(8.5,0.0,1.5)</td>
<td>4.05</td>
<td>11.88</td>
<td>4.31</td>
</tr>
<tr>
<td>Ant.4-(8.5,4.0,1.5)</td>
<td>3.43</td>
<td>9.67</td>
<td>3.96</td>
</tr>
<tr>
<td>Occupied room</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ant.1-(0.0,0.0,1.5)</td>
<td>6.14</td>
<td>24.76</td>
<td>8.12</td>
</tr>
<tr>
<td>Ant.2-(0.0,4.0,1.5)</td>
<td>6.12</td>
<td>12.18</td>
<td>1.98</td>
</tr>
<tr>
<td>Ant.3-(8.5,0.0,1.5)</td>
<td>3.99</td>
<td>11.74</td>
<td>4.32</td>
</tr>
<tr>
<td>Ant.4-(8.5,4.0,1.5)</td>
<td>3.71</td>
<td>9.92</td>
<td>3.57</td>
</tr>
</tbody>
</table>

Next, we proceed with the extraction of the bit error rate.

### 2.7.3 Bit Error Rate Calculations in Multi-Ricean Channels

An important parameter in any digital communication system is the bit error rate (BER). Using the extracted $K$-factors, we are in a position to predict BER for specific coding systems that use coherent demodulation and maximal-ratio combining (MRC) in the given indoor environment. Specifically, we follow [82,84] to derive an alternative BER expression over the multi-Ricean fading channel that is easier to use for our calculations

$$P_2 = \frac{1}{\pi} \prod_{l=1}^{m} \left( \frac{\sin^2 \phi_l}{\sin^2 \phi_l + g^2 b} \right) \exp \left[ \frac{-K_l g^2 b}{\sin^2 \phi_l + g^2 b} \right] d\phi_1 \cdots d\phi_m. \tag{2.12}$$
Here, \( g \) is associated with the modulation type, being \( g = 0.5, 1 \) for coherent BFSK and BPSK, respectively. Also, \( K_l = \frac{A_l^2}{\sigma_l^2} \) corresponds to the \( K \)-factor of the \( l \)th antenna, with \( m \) denoting the total number of antennas, and \( \bar{\gamma}_b = \frac{2\sigma_e^2E_b}{N_0} \) with \( \bar{\gamma}_b \) being the signal-to-noise ratio (SNR) per bit. As usual, \( A_l \) is the peak amplitude of the dominant signal and \( \frac{E_b}{N_0} \) is the SNR. Based on (2.12), we present in Figs. 2.16-2.18 the BER over a (multi) Ricean fading channel for one, two, and four antennas, respectively. Fig. 2.16 gives the BER for Ricean fading channels with \( K = 2 \) and 8. We observe that as \( K \) is increased from 2 to 8, the BER is reduced by a factor of 100 (with \( \bar{\gamma}_b = 6 \)dB). As already noted, placing the transmitting antenna at \((0.0, 0.4, 1.5)\) versus \((8.5, 4.0, 1.5)\), implies a change in \( K \) from 1.86 to 9.67, and therefore a BER reduction by a factor of 100.

So far, we only varied the height of the transmitting antennas to improve BER. In fact, as is known, multiple antennas can also improve the performance of the communication systems. In the following section, we explore this situation.

### 2.7.4 Bit Error Rate for Multiple-Antennas Over a Multi-Ricean Channel

In this section, we considered the influence of multiple-antennas on BER. First, the impact of multiple antenna locations is investigated. Fig. 2.17 presents a comparison of BER for a pair of antennas placed at \((0.0, 0.0, 1.5)\) and \((8.5, 0.0, 1.5)\) (denoted as Antenna 1 and Antenna 3, respectively) vs. another antenna pair located at \((0.0, 4.0, 1.5)\) and \((8.5, 4.0, 1.5)\) (denoted as Antenna 2 and Antenna 4, respectively). The figure indicates that antenna location can have significant impact on
system performance (i.e. BER). Specifically, it is demonstrated that the pair (Antenna 1, Antenna 3) gives a BER improvement by a factor of 100 as compared to the transmitting (Antenna 2, Antenna 4) pair (with $\bar{\gamma}_b = 6$dB for BPSK modulation).

The impact on BER when using additional antennas, specifically with 2 and 4 transmitting antennas, is depicted in Fig. 2.18 (based on the data at 81 receiving locations in Fig. 2.6). The curves in Fig. 2.18 clearly show a significant improvement (as predicted) when additional transmitting antennas are used in conjunction with binary frequency shift keying (BFSK) and binary phase shift keying (BPSK). Specifically, BER is reduced by a factor of $10^5$ (with $\bar{\gamma}_b = 6$dB) for BPSK modulation when 3 additional transmitting antennas are employed.

Based on the results of this chapter, we can remark that

1. we can determine an optimal transmitting location to achieve the highest data rate across a site-specific propagation channel, and

2. we can improve data capacity using multiple antennas in the system.

This work has also suggested that we can provide design guidelines for site-planning in wireless systems using rigorous tools.

2.8 Conclusions

We presented a validation of rigorous method for predicting indoor channel parameters at 2.4GHz. The basis of the method was to decompose the structure into repeatable sections. In doing so, only unique repeatable blocks need to be stored, and as a result, there is a significant memory and CPU reduction. Having modeled a realistic classroom, we proceeded to extract statistical parameters (i.e. $K$-factor)
Figure 2.16: Bit error rate (BER) calculation for Ricean fading channels with $K = 2, 4$ and $8$; that is the case for single transmitter single receiver; ASK denotes amplitude shift keying and FSK denotes frequency shift keying.
Figure 2.17: Comparison of BER for the pair (Antenna 1, Antenna 3) as compared to the (Antenna 2, Antenna 4) pair; Antennas 1-4 are located at (0.0, 0.0, 1.5), (0.0, 4.0, 1.5), (8.5, 0.0, 1.5) and (8.5, 4.0, 1.5), respectively; As before, 81 receiving locations were used in the statistical modeling to extract BER; ASK denotes amplitude shift keying and FSK denotes frequency shift keying.
Figure 2.18: Bit error rate (BER) comparison for a single transmitting antenna, 2 transmitting antennas, and 4 transmitting antennas over the considered fading channel; for the single antenna, the transmitter was at \((0.0, 0.0, 1.5)\); for the 2 antennas, the transmitters were at \((0.0, 0.0, 1.5)\) and \((8.5, 0.0, 1.5)\); for the 4 antennas, the transmitter locations were at \((0.0, 0.0, 1.5)\), \((0.0, 4.0, 1.5)\), \((8.5, 0.0, 1.5)\) and \((8.5, 4.0, 1.5)\); As before, 81 receiving locations were used in the statistical modeling to extract BER; ASK denotes amplitude shift keying and FSK denotes frequency shift keying.
to be used for BER evaluations. We presented an approach to determine the performance of digital communications (based on BER) using an EM full-wave analysis and a statistical model for a site-specific indoor environment. Based on the understanding of the propagation channels, we proceed to optimize the transmitting location of the antennas for a higher data rate.

In this chapter, beside varying the height of the transmitting antennas to achieve maximum performance, we demonstrate that the optimal position of the transmitting antenna can be determined via our proposed procedure. Similar ideas have also be extended to the deployment of multiple antennas for high-speed communication systems.
CHAPTER 3

MIMO SYSTEM CAPACITY USING RIGOROUS METHODS

3.1 Introduction

Improvements in capacity and bit error rate (BER) have drawn considerable attention toward multiple antennas systems, also known as multiple input multiple output (MIMO) systems [85–106]. Such systems exploit the spatial properties of the multipath channel, and have demonstrated the potential to significantly increase channel capacity in multipath-rich indoor environments. As already shown [85,86,98], MIMO offers possibilities for enhancing the performance of a wireless communication system. While channel coding and array signal processing are key components to successfully implement a MIMO system, the propagation channel plays an important role as well. Previous theoretical work was based on a more simplified Rayleigh fading [86,107] which assumes that there is no dominant signal present. However, recent investigations [89,97] demonstrate that under special cases (called “keyholes”), capacity estimation based on independent Rayleigh-fading assumptions is no longer valid. Therefore, a more accurate channel model is appropriate for a more precise estimation of channel statistics.
Measurements [108–118] or simulations [40–47, 119, 120] have been carried out to obtain more realistic models of the channel. However, as is well known, measurements can be expensive and tedious. Consequently, simulations become obvious candidates for indoor channel modeling, due to cost and flexibility. Ray-tracing [40–43, 120] and full-wave finite difference time-domain (FDTD) methods [44–47, 119] are among the commonly used methods. Ray-tracing approaches, although fast, are not well suited for penetrable/dielectric structures (with many multipath) as is the case with indoor propagation channels. On the other hand, FDTD is computationally demanding and not practical for 3D representation of an entire classroom or apartment.

In this chapter, we again propose the use of the array decomposition-fast multipole method (AD-FMM) for indoor channel modeling [76, 106, 121–124]. AD-FMM method employs domain decomposition in conjunction with a rigorous finite element-boundary integral (FE-BI) method and has been successfully used in characterizing complex indoor channels. For example, it has already been used for full-wave analysis of a computational domain as large as $50\lambda \times 25\lambda \times 25\lambda$.

As can be realized, a rich set of data (i.e. for the power delay profiles (PDPs)) must be collected for statistical modeling. Using these statistics, channel parameters (such as the $K$-factor for Ricean distribution) can be extracted directly from PDPs to represent the propagation channel associated with a site-specific indoor environment. As such, one can obtain the complementary cumulative distribution function (CCDF) of channel capacity and subsequently explore multipath effects in a MIMO setting. The results can be subsequently used to optimize antenna location, spacing, and height.
This chapter is organized as follows. Section 3.2 presents the simulation setup and statistical extraction process. The extraction of channel capacity and related discussions are given in Section 3.3. Section 3.4 concludes the chapter.

3.2 Small-Scale Fading Statistics for MIMO systems

We proceed now with the full-wave analysis of a classroom based on the same approach for the purpose of extracting related statistics associated with the propagation within the classroom as depicted in Fig. 2.4. Again, we explore the two examples of interest, namely (1) an empty room and (2) the same room occupied with 20 chairs (to be called “furnished room”). The dimensions of the room are depicted in Fig. 2.4, with the detailed floor plan displayed in Fig. 3.1. The room is of length 9.0m, width 4.5m and height 2.1m. The walls, floor, and ceiling are 7cm thick having a relative dielectric constant ($\epsilon_r$) 4.0 and conductivity 2.0S/m. Each of the 9.0m $\times$ 2.1m walls has 5 windows 0.9m $\times$ 0.7m in size (see Fig. 2.4).

Three types of repeatable blocks were used to model the repeatable wall blocks using AD-FMM method. We will refer to these block types as Type I, Type II (for the walls) and Type III (for the ceiling/floor). Type I was of size of 0.9m $\times$ 0.7m $\times$ 0.07m, Type II was of size of 0.7m $\times$ 0.9m $\times$ 0.07m and Type III was of size of 0.9m $\times$ 0.9m $\times$ 0.07m. Using these blocks, 50 Type I, 30 Type II and 100 Type III blocks were used to model the walls and ceiling/floor. For the chairs, we used a fourth block (Type IV) repeatedly as needed to form the rows and columns. In doing so, only 304,000 unknowns, requiring 5 Gbytes in memory, were needed to model the entire classroom with furniture. For the propagation analysis, antennas were placed at one corner of the room (see Fig. 3.1) transmitting a CW (continuous wave) Gaussian
pulse of bandwidth 500 MHz at 2.4 GHz. As noted already, two simulations were conducted, one for the empty room and another for the same room with 20 chairs.

As described in the earlier chapter, multipath occur via the interactions among walls, furniture, or other objects within the propagation path. As a result, signals arrive at the receiver through several paths, causing fading. This fading can be statistically obtained from the PDPs by developing a cumulative distribution function (as demonstrated in Section 2.5). In doing so, we can examine the best-fit distribution for the observed histogram (by means of the maximum likelihood estimation (MLE) method). Here, we chose the Ricean distribution for fitting the data because Ricean distribution has shown to describe the fading better in this
study (see Fig. 2.8). To create the Ricean fitting, we performed a null hypothesis testing, \( H_0 \) : (Observed data = fitted Ricean) versus the alternative hypothesis testing, \( H_A \) : (Observed data \( \neq \) fitted Ricean) using the Kolmogorov-Smirnov (KS) goodness-of-fit test. Furthermore, to ensure a good fit within a reasonable tolerance, the significant level was kept within 5\%. The obtained statistics via this procedure were subsequently used to mitigate the fading effects.

To combat multipath effects, various means were proposed [121–124]. Therefore, in this context, we only investigated the implementation of MIMO systems. Specifically, we explored a system with \( N \) transmitting antennas located at a corner of the room and a corresponding row of \( M \) receiving antennas placed opposite corner, as depicted in Fig. 3.1. For simplicity, each antenna was modeled as an infinitesimal dipole (i.e. no coupling among antenna elements was assumed). Clearly, several multipath exist and from previous studies [121–124], these multipath can be characterized by a Ricean distribution (with a statistical \( K \)-factor). The statistical \( K \)-factor can be extracted from the power delay profile (PDP), where PDP is given by \( \frac{1}{2}(\epsilon |\bar{E}|^2 + \mu |\bar{H}|^2) \), with \((\bar{E}, \bar{H})\) being the total electric and magnetic field vectors, respectively. For MIMO systems, we have \( N \) transmitters and \( M \) receivers, resulting in a \( N \times M \) coupling matrix to be referred to as \( \mathbf{K} \) (with each matrix element describing the transmitting element to receiving element \( K \)-factors) of the form

\[
\mathbf{K} = \begin{bmatrix}
K_{11} & K_{12} & \cdots & K_{1N} \\
K_{21} & K_{22} & \cdots & K_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
K_{M1} & K_{M2} & \cdots & K_{MN}
\end{bmatrix}.
\]  

(3.1)

In (3.1), \( K_{ji} \) is the \( K \)-factor of the Ricean distribution describing the multipath between the \( i \)th transmitter and \( j \)th receiver within the propagation channel.
Having extracted the multipath statistics provided by (3.1), we next proceed to calculate channel capacity.

### 3.3 Channel Capacity Evaluation of MIMO Systems Via AD-FMM

To evaluate channel capacity in a MIMO system, it is necessary to carry out a large number of propagation simulations. Since each simulation requires extensive computational resources, it is highly intensive to carry out separate re-calculation for each configuration (antenna arrangement, geometry set-up, furniture arrangement etc.). Instead, we propose a Monte-Carlo procedure using the results from a single configuration analysis. The proposed Monte-Carlo procedure uses a Ricean random number generator (which takes the corresponding K-factor values from (3.1)) to evaluate 10000 different random multipath scenarios. As such, we can collect 10000 different realizations and each realization is defined by the $P$. The latter is equivalent to $H^H H$ where $H$ is the channel matrix with the superscript $H$ denoting the Hermitian transposition. The matrix $H$ describes the signal propagation among $N$ transmitters and $M$ receivers, and is explicitly given by the matrix

$$
H = \begin{bmatrix}
    h_{11} & h_{12} & \cdots & h_{1N} \\
    h_{21} & h_{22} & \cdots & h_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{M1} & h_{M2} & \cdots & h_{MN}
\end{bmatrix}, \quad (3.2)
$$

where $h_{ji}$ is the transfer function of the transmit-receive signal for channel between the $i$th transmitter and $j$th receiver. Explicitly, $h_{ji}$ takes the form:

$$
h_{ji} = \sum_{l=0}^{P-1} A_{l,j} \exp(\sqrt{-1} \phi_{l,j}) \delta(t - \tau_{l,j}), \quad (3.3)
$$
where $P$ is the number of multipath components with $A_{l,j}$, $\tau_{l,j}$ and $\phi_{l,j}$ representing the amplitude, time delay and phase shift, respectively (of the $i^{th}$ multipath component at $j^{th}$ receiver due to $i^{th}$ transmitter). In previous work [86,107], $A_{ji}$ of the multipath components were modeled as Rayleigh distributed random variables. However, studies have also suggested that in those cases where a dominant line-of-sight (LOS) exists, capacity estimation based on the Rayleigh-fading assumptions is no longer valid. This motivates the use of rigorous AD-FMM for statistical extraction.

Having the matrix $H$, one can compute the capacity using Shannon’s expression [86]

$$C = \log_2 \det \left( I + \frac{\rho}{N} HH^H \right),$$

(3.4)

where $\rho$ denotes the signal-to-noise ratio (SNR), $N$ is the number of transmitting antennas, $I$ is the identity matrix, with the superscript $H$ denoting Hermitian transposition.

As such, and using the Monte-Carlo procedure, we can generate a histogram of the 10000 channel realizations. This histogram is known as the cumulative distribution function (CDF) of the capacity. Typically, the complementary form of the CDF (equal to $1 - \text{CDF}$ and referred to herewith as complementary cumulative distribution function (CCDF)) is plotted. Such plots provide a relation of the channel capacity due to variations of the multipath within the channel. As a result, we used the CCDF of the channel to gauge the effect of spacing among antennas.

Many studies have already demonstrated the increased capacity aided by multiple antennas in a Rayleigh fading environment. However, few have considered the case of a Ricean fading channel. Here, we employed the AD-FMM method to extract the
capacity of $2 \times 2$ and $4 \times 4$ MIMO systems (see Figs. 3.1 and 3.4), corresponding to an empty and a furnished room. It is important to discuss the computational requirements for these MIMO settings, specifically to model the aforementioned rooms. AD-FMM required 1.86 Gbytes and 5.11 Gbytes of shared memory for the empty and furnished rooms, respectively. Without the adaption of AD-FMM, the standard FE-BI method would require a projected 378 Gbytes of memory for the empty room, and even more memory requirement (projected at 458 Gbytes) for the room with furniture. Of course, even with present available super-computers, solving problems of this dimension, is not possible for a straightforward application of standard full-wave methods (namely, FDTD, MoM, and FEM). With this significant memory reduction via AD-FMM, rigorous indoor channel propagation analysis can be carried out within practical time frames.

Of particular interest is the effect of antenna locations among the receiving antennas, and to do so, we considered a $2 \times 2$ MIMO system (see Fig. 3.1). Calculations of the transfer matrix with 4 sets of antenna pair locations of $(7.8m,3.8m,1.5m)$ & $(7.6m,3.8m,1.5m)$, $(7.8m,3.8m,1.5m)$ & $(7.2m,3.8m,1.5m)$, $(7.8m,3.8m,1.5m)$ & $(6.8m,3.8m,1.5m)$, and $(7.8m,3.8m,1.5m)$ & $(6.4m,3.8m,1.5m)$ were performed, with the resulting CCDFs given in Fig. 3.2 (for an empty room) and Fig. 3.3 (for a furnished room). We observe that antenna pair locations of $(7.8m,3.8m,1.5m)$ & $(7.6m,3.8m,1.5m)$ and $(7.8m,3.8m,1.5m)$ & $(7.2m,3.8m,1.5m)$ lead to the best capacity among the rest of the MIMO set-ups for the empty and furnished rooms, respectively. That is, the chairs in the furnished room implied a different set of antenna locations for optimal capacity. As can be expected, the addition of chairs alters the characteristics of the propagation channel. Since the chairs increase the number
of the multipath components, a different propagation characteristics are observed. This particular conclusion was drawn from previous studies using AD-FMM [123] and experimentally [80]. That is, objects such as chairs, tables, computers, etc. can obstruct propagation paths, giving rise to more multipath via diffractions and reflections. As a result, the statistics of the propagation channel tends to approach that of Rayleigh distribution, implying that there is no dominant signal. We would expect that antenna spacing would play a less significant role in an environment with many multipath. Indeed, by referring to Figs. 3.2 and 3.3, we can clearly see that varying the antenna pair locations between (7.8m, 3.8m, 1.5m) & (7.6m, 3.8m, 1.5m) and (7.8m, 3.8m, 1.5m) & (6.4m, 3.8m, 1.5m) has less effect for the furnished room (a difference of 1.1 bps/Hz at CCDF=0.5) than the empty room (a difference of 2.7 bps/Hz at CCDF=0.5). These results also suggest that capacity estimation based on the Rayleigh-fading assumptions is no longer valid unless a rich multipath environment with no dominant signal exists. This in itself, motivates the need for a rigorous simulation method for indoor channel characterizations.

To examine the channel capacity when a larger number of antennas are used in a Ricean environment, we compared $2 \times 2$ and $4 \times 4$ MIMO systems. For this case, the transmitting and receiving antenna pairs were placed as shown in Fig 3.1, with a 0.2m spacing between antennas. As for the $4 \times 4$ MIMO setting, the layout is depicted in Fig. 3.4. In this context, the same procedure (AD-FMM and statistical methods) was carried out. The extracted CCDFs for the $2 \times 2$ and $4 \times 4$ MIMO systems are compared in Fig. 3.5. As expected, the capacity increases by using the $4 \times 4$ MIMO system. More specifically, the $4 \times 4$ MIMO system has twice the capacity of a $2 \times 2$ system, and this is due to the exploitation of channel spatial properties.
Figure 3.2: Effect of antenna locations for $2 \times 2$ MIMO system in an empty room.
Figure 3.3: Effect of antenna locations for $2 \times 2$ MIMO system in a furnished room.
We note that all these simulations were ran on the Itanium dual-processor 900MHz PC clusters at the OSC. As can be realized, the simulation time (based on AD-FMM) for the empty and furnished rooms, were only 0.7 hour and 5.0 hours, respectively. Due to the memory requirements, standard simulations using FE-BI method could not be possible for this application.

3.4 Conclusions

We presented the application and validation of a full-wave rigorous method for predicting indoor channel parameters at 2.4 GHz. The basis of the ADM is to decompose the structure into repeatable sections, and by doing so, only unique repeatable
Figure 3.5: Channel capacity comparison for the $2 \times 2$ and $4 \times 4$ MIMO systems in an empty room.
blocks need be stored. As a result, significant 10-fold memory and CPU reductions were achieved, making ADM and its FMM enhanced version suitable for full wave simulations of furnished rooms and classrooms for channel characterization.

We demonstrated use of AD-FMM method in extracting a key statistical parameter (K-factor of the Ricean distribution) for an indoor channel in a MIMO setting. These statistics were subsequently used to compute the CCDF of the channel capacity in conjunction with a Monte-Carlo procedure. Using these CCDFs, we considered the effect of antenna spacings for MIMO systems placed in an empty and furnished room. As expected, the furniture in the room have great impacts in the performance and capacity of MIMO systems for all the antenna locations except for (7.8m,3.8m,1.5m) & (7.6m,3.8m,1.5m) antenna locations, pointing to the need for rigorous models such as the AD-FMM.
CHAPTER 4

60-GHZ INDOOR CHANNEL CHARACTERIZATION USING RAY-TRACING METHODS

4.1 Introduction

Increasing demand of real-time high-speed applications calls for wireless local area network (LAN) operating in the 60 GHz band as part of the 4th generation (4G) system. The 60 GHz band has spiked great interest [64, 125–129] because of its large bandwidth (7 GHz) allocated for future dense wireless local communications, particularly as relates to large wireless LAN bridges, and wireless high-quality video-conferencing. To establish such links, wireless systems which exploit time, frequency and spatial multiplexing, may be required. Design of these communication systems involves space-time coding, adaptive antennas and rake reception which rely strongly on the characterization of the propagation channel. Previous work in channel characterizations at these millimeter (mm) wave frequencies have depended on measurements [63–67]. However, measurements can be expensive (especially in the mm-wave band) as compared to electromagnetic (EM) modeling approaches. Since rigorous numerical methods are ruled out due to the very short wavelength at mm-waves, we consider
high-frequency asymptotic approaches such as ray-tracing (RT) methods for modeling the channels. RT methods have the capability to solve electrically large problems relatively fast and, as such, they become an obvious candidate for the extraction of channel parameters. In this chapter, we compare the channel parameters based on the RT model with in-house collected measurements, and measurements obtained from [63]. Subsequently, we provide results for the fading statistics of the received power in two typical indoor propagation channels, namely within a room and in a hallway.

The chapter is organized as follows. The next section presents the validation of the ray-tracing model using in-house measurement results obtained in the 2-3 GHz band. Section 4.3 describes the EM modeling of the room and hallway, and the simulation setup. Extraction of the channel parameters and modeling of the fading statistics are presented in Section 4.4. Section 4.5 concludes the chapter.

4.2 Validation of the Ray-Tracing Model with Measurements

The numerical electromagnetic code-basic scattering code (NEC-BSC) [130], which is based on 3-dimensional (3-D) ray-tracing technique, utilizes the uniform asymptotic concepts formulated in terms of the Uniform Geometrical Theory of Diffraction (UTD) [131,132]. As such, UTD is ideal for understanding the high frequency response of signal in a complex environment whereby the basic structural features (that are crucial for accuracy) of that complex environment are necessary for modeling. In doing so, this allows for the use of ray optical techniques for obtaining the incident, reflected and diffracted rays, contributed from these various basic structures. As a result, the reflected and diffraction fields are subsequently determined using the UTD
solutions which consist of the individual rays that are summed with the geometrical optics in the far zone of the scatterer. As we know, the rays from a given scatterer tend to interact with other nearby objects, resulting into higher-order rays. As such, NEC-BSC was built to take care of all these high-order interactions, but not all high order contributions are significant. Therefore, one can also choose to include only dominant contributions in NEC-BSC. Given all these, NEC-BSC is appropriate in this 60 GHz propagation study and it is employed to obtain power delay profiles (PDPs) for the indoor propagation channel. As a first step, we proceed to validate the ray-tracing model with measurements for the indoor propagation channel considered in this chapter.

4.2.1 Measurement Setup

The measurement setup consisted of a network analyzer (i.e. Agilent E8362B), a pair of 180° hybrid couplers and a pair of identical bow-tie antennas (denoted as Antenna1 and Antenna2). The bow-tie antennas were designed to have a center frequency of 2.5 GHz, with flare angle 45° and 1 GHz bandwidth sufficient for this measurement. An empty room was chosen (see Fig. 4.1) whose dimensions are depicted in Fig. 4.2. Specifically, the room is of length 7.72m, width 5.84m and height 2.82m. Antenna 1, operating as a transmitter, was positioned at (0.94m,0.76m) and at a height of 2.24m. Antenna 2, serving as a receiver, was placed at 18 different locations inside the room (standing at the height of 1.12m) for measurements. The detailed position of these 18 locations is depicted in Fig. 4.2. For consistency, four measurements were taken at each of these locations and the average of these four measurements was used as the result. For each measurement, a total of 1601 frequency
points (i.e. \( S_{21} \)) between 2 GHz and 3 GHz were used, resulting in a frequency step of 0.625 MHz. This frequency resolution implied a maximum excess delay of about 1600 nsec and a temporal resolution of 1 nsec (because of the 1 GHz bandwidth). We remark that a signal-to-noise ratio (SNR) of at least 20dB was maintained throughout all measurements (via averaging during data sampling).

### 4.2.2 Simulations

For our simulations, the NEC-BSC was used. We computed the response at the same 1601 continuous wave (CW) tones evenly spaced between 2 GHz and 3 GHz as done with the measurements. For these calculations, the direct and reflected rays up to tenth order (from the walls, ceiling and floor) were included. The walls, floor, and ceiling were characterized by relative dielectric constant of 6.0 and conductivity 2.0 \( S/m \) whereas the walls were of thickness 14.5cm. Both the transmitting and receiving antennas (i.e. Antenna 1 and Antenna 2) were modeled in BSC as having omni-directional radiation pattern. We remark that the simulation time of each location was approximately 139 mins using a 1.6 GHz central processing unit (CPU) machine.

### 4.2.3 Validation Results

As is expected, one-to-one mapping of indoor propagation measurements to simulations is rarely achieved. As such, one can explore a stochastic way of validating the measurement and simulation data [74]. Specifically, we compared the time-domain multipath channel parameters such as mean excess delay and root mean square (RMS) delay spread [78]. These parameters are useful in describing the overall characteristics of the multipath profile and are essential in developing design guidelines for digital
Figure 4.1: Photograph of the empty room where the measurements were conducted. The inset shows some of the measuring locations.
Figure 4.2: The positions of the 18 measuring locations and the transmitting location, all within the classroom of dimensions, length 7.72m, width 5.84m and height 2.82m.
wireless communication systems. These channel parameters are easily extracted from
the power delay profiles (PDPs). To obtain the PDP at a given receiver location, the
1601 CW tones are transformed to the time domain via an inverse fast Fourier trans-
form (IFFT) procedure. Therefore, each of the 18 measuring locations (see Fig. 4.2)
is associated with a PDP and a set of multipath channel parameters. Of particular
importance is the RMS delay spread ($\sigma$), which equals to the square root of the sec-
ond moment of the PDP [78]. This is an indicator of the maximum data rate in the
wireless channel and is also directly related to the performance degradation caused by
inter-symbol interference (ISI). Given the importance of RMS delay spread, we used
this parameter for comparing the measured and calculated data. As 18 measuring
locations were considered here, we built a cumulative distribution function (CDF) for
the RMS delay spread values. Fig. 4.3 shows the measured and simulated RMS delay
spread CDFs and its associated probability density functions (PDFs) are depicted in
Fig. 4.4. Clearly, there is a good agreement between measurements and simulations,
indicating that the BSC can be employed for predicting the multipath channel para-
eters. From Fig. 4.4, we can conclude that most of the locations have RMS delay
spread between 40 nsec and 60 nsec. Next, we proceed with a study at 60 GHz based
on the NEC-BSC.

4.3 Modeling of Room and Hallway

For our 60 GHz propagation studies, of particular interest was the effect of wall
configuration on the channel parameters and the fading statistics. Thus, we consid-
ered two configurations: (1) a room and (2) a hallway. The dimensions of the room
are depicted in Fig. 4.5 and the dimensions of the hallway are depicted in Fig. 4.6.
Figure 4.3: Comparison of measured and simulated RMS delay spread CDFs in the empty room; The solid line denotes the RMS delay spread obtained from our simulations; The dotted line represents the measured RMS delay spread.
Figure 4.4: Comparison of measured and simulated RMS delay spread PDFs in the empty room; The circle denotes the RMS delay spread obtained from our simulations; The square represents the measured RMS delay spread.
The room has length 8.4m, width 7.0m and height 4.3m, whereas the hallway has length 54.7m, width 2.9m and height 4.3m. The walls, floor, and ceiling are 14.5cm thick characterized by a relative dielectric \((\epsilon_r) 4.22\) and conductivity 0.02 \(S/m\). For propagation analysis, we chose a horn antenna as the transmitter with a theoretical half power beamwidths (HPBW) of 12° in azimuth and 9.5° in elevation. The receiving antennas were considered to have an omni-directional radiation pattern. We remark that all receiver positions had a line-of-sight (LOS) path to the transmitter. Specifically, four receiving locations for both the room (see Fig. 4.5) and hallway (see Fig. 4.6), namely R11-R14 and R21-R24 were sampled. At these locations, channel parameters and fading statistics were extracted as described in Section 4.4.

For the simulations, the NEC-BSC was set to analyze the propagation response using 1601 continuous (CW) tones evenly spaced between 59 GHz and 61 GHz, which resulting in a frequency sweep with 1.25 MHz steps. As a result, the frequency resolution had a maximum excess delay of about 166.66 nsec and a temporal resolution of 500 psec (because of 2 GHz bandwidth). In the simulations, the direct and reflected rays up to tenth and seventh order from the walls, ceiling and floor, were included for the room and hallway, respectively. The reasons for the chosen order are twofold. Firstly, the accuracy of these simulations needed for this study was satisfied. Secondly, we wanted to keep our simulation times within 3 hrs. Here, our interest is the extraction of the multipath channel parameter (i.e. RMS delay spread). As such, the 1601 CW tones are transformed to time domain to obtain the channel response (i.e. PDP) at each receiver location. We note that the simulation times for each receiving location are approximately 67 mins and 142 mins for the room and hallway, respectively, using a 1.6 GHz CPU machine.
Ceilings lifted up for illustration

Note: All dimensions are in meter.

Figure 4.5: 3-D view of the room and its floorplan used for the 60 GHz simulations.
Figure 4.6: 3-D view of the hallway and its floorplan.

Note: All dimensions are in meter.
Table 4.1: RMS delay spread of room and hallway as shown in Fig. 4.5 and Fig. 4.6, respectively.

<table>
<thead>
<tr>
<th>Rx location</th>
<th>Room</th>
<th>σ [ns]</th>
<th>Hallway</th>
<th>σ [ns]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R11-(7.4,6.0,1.6)</td>
<td>31.20</td>
<td>R21-(44.2,10.1,1.6)</td>
<td>58.15</td>
<td></td>
</tr>
<tr>
<td>R12-(1.0,6.0,1.6)</td>
<td>24.85</td>
<td>R22-(35.7,10.1,1.6)</td>
<td>65.32</td>
<td></td>
</tr>
<tr>
<td>R13-(7.4,1.0,1.6)</td>
<td>51.28</td>
<td>R23-(27.4,10.1,1.6)</td>
<td>51.88</td>
<td></td>
</tr>
<tr>
<td>R14-(4.2,3.5,1.6)</td>
<td>36.26</td>
<td>R24-(54.2,10.1,1.6)</td>
<td>57.44</td>
<td></td>
</tr>
</tbody>
</table>

4.4 Channel Parameter and Fading Model

Next, we proceed to extract the multipath channel parameter (i.e. RMS delay spread) at 60 GHz. Table 4.1 shows the RMS delay spread ($\sigma$) at the various receiving locations for both the room and the hallway. When the receiving antenna is placed at different locations, the delay spread ranges from 24.85 nsec to 51.28 nsec for the room and 51.88 nsec to 65.32 nsec for the hallway. The simulated delay spreads are in agreement with the measurement results in [63]. In the case of [63], the delay spreads for indoor 60 GHz channels range from 15 nsec to 45 nsec for small rooms and between 30 nsec and 70 nsec for large indoor environments. This also implies that the ray tracing method can be used to predict the multipath channel parameters at the mm-wave frequencies.

As is well-known, indoor propagation involves interactions among furniture, walls or other objects. Because of these multipath, signals arrive at the receiver with different phases, causing fading. This fading can be obtained statistically from the PDPs by first developing a cumulative distribution function (CDF) based on the probability of receiving energies above a pre-determined threshold level. Next, we look
for the best-fit distribution for the observed CDF (by means of maximum likelihood estimation (MLE)). In this analysis, we chose the Weibull distribution (which has also been used for ultra-wide band indoor propagation) for fitting the data. This is because we tested Rayleigh and Ricean distributions and both distributions did not fit the data better than the Weibull distribution. The latter has a probability density function defined as

\[
p(r) = \begin{cases} \frac{b a^{-b} r^{b-1} \exp \left( -\frac{r}{a} \right)^b}{\Gamma(b)} & ; \quad 0 \leq r \leq \infty \\
0 & ; \quad r < 0 \end{cases},
\]

(4.1)

where \( a \) and \( b \), respectively, are the scale and the shape parameters chosen to fit the simulations.

To check the fitting of the observed and estimated Weibull data, we performed a null hypothesis testing, \( H_0 : (\text{Observed data} = \text{fitted Weibull}) \) versus the alternative hypothesis \( H_A : (\text{Observed data} \neq \text{fitted Weibull}) \) by using the Kolmogorov-Smirnov (KS) goodness-of-fit test. To ensure a good fit within a reasonable tolerance, the significant level was kept within 5 \%. In both the room and the hallway studies, it is clearly shown in Fig. 4.7 (i.e. R11-R14) and Fig. 4.8 (i.e. R21-R24) that the CDFs at receiving locations have a good agreement with the Weibull distribution.

4.5 Conclusion

Based on the 3D ray-tracing method, we extracted statistical parameters (i.e RMS delay spread) for indoor site-specific environments of different configurations. We found that the fading statistics of these indoor environments were characterized by a Weibull distribution. This is particularly due to the large 2 GHz operating bandwidth, where fading statistics are no longer obeying Rayleigh or Ricean distributions. As can be understood, accurate prediction of such statistics is vital in determining the
Figure 4.7: Cumulative distribution function (CDF) computed from the received power over mean power in Fig. 4.5. The dots are CDF of the simulations of received power over mean power at R11-R14 and the depicted solid lines come from the best-fitted Weibull distribution.
Figure 4.8: Cumulative distribution function (CDF) computed from the received power over mean power in Fig. 4.5 and Fig. 4.6. The dots are CDF of the simulations of received power over mean power at R21-24 and the depicted solid lines come from the best-fitted Weibull distribution.
channel capacity and this has been shown in [123]. In conclusion, it has been demonstrated that the ray-tracing methods can be used for channel parameter extractions, particularly at 60 GHz band.
CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 Summary and Conclusions

Accurate modeling of the propagation channel is increasingly important in wireless communications, particularly in the area of dense and high speed communications. The work presented in this dissertation contributes to this aspect by introducing full-wave electromagnetic (i.e. the array decomposition-fast multipole method (AD-FMM) [123]) methodology and revisiting ray-tracing method (i.e. numerical electromagnetic code-basic scattering code (NEC-BSC) [77]) for this purpose. The former employs repeatable components for practical modeling indoor structures (such as walls, furniture, etc) which are repeatable in nature. This repeatability was exploited to reduce computational memory requirements for practical realization of indoor channels using full-wave methods.

In this dissertation, we chose a classroom example depicted in Fig. 2.4 to demonstrate our approach. Based on this representative example, we extracted important channel parameters in wireless communications, namely, the mean excess delay and the root mean square (RMS) delay spread. Another vital statistical component pertaining to the fading is the fading statistics. In this regard, we concluded that the
Ricean distribution is the best statistical channel model for describing the behavior of the multipath fading in the indoor environment. That is, for narrowband communication channels, the fading statistics obey Ricean distribution when there is a dominant signal among all the multipath signals (as one would expect). Furthermore, based on the rigorous method demonstrated in this dissertation, we were able to provide the statistical $K$-factor, pertaining to the ratio of the dominant signal to the multipath signals.

Having extracted the fading statistics (i.e. $K$-factor), we proceeded to evaluate system performance. Specifically, we considered 2 binary modulation schemes, namely the binary phase shift keying (BPSK) and the frequency shift keying (BFSK) over a Ricean fading channel and computed the bit error rates (BERs). Based on BER calculations, we proposed ways of improving system performance (such as determining antenna height for optimal transmission and deploying multiple antennas for dense communication links). Specifically, we found out that placing transmitting antenna at $(0.0,0.0,1.5)$ instead of the other locations (i.e. $(0.0,4.0,1.5)$, $(8.5,0.0,1.5)$ and $(8.5,4.0,1.5)$) of Fig 2.6 gave the best BER performance. Furthermore, we concluded that having 2 antennas transmitting at $(0.0,0.0,1.5)$ and $(8.5,0.0,1.5)$ had the best BER performance than any other 2 combinations. As one would expect, placing more antennas would improve the performance and this had also been observed in this study (see Fig 2.18).

For the 4th generation (future) systems operating at 60 GHz, we revisited the ray-tracing method since rigorous numerical methods are ruled out at such short wavelength. We found that at 60 GHz, the channel fading is different and concluded that a Weibull distribution is more appropriate. This is particularly due to the large
2 GHz operating bandwidth, where the fading statistics are no longer Rayleigh or Ricean distributions as like many narrowband communication systems had assumed.

5.2 Future Work

There are 2 interesting areas which can be addressed beyond this work. One of them refers to a hybridization of AD-FMM and BSC to study multiple input multiple output (MIMO) for ultra-wide band (UWB) systems. The second refers to the study of MIMO configurations in 4G systems. Of particular interest are MIMO systems based on orthogonal frequency division multiplexing (OFDM). The potential of these systems for high data capacity is of great interest.

5.2.1 Channel Model Based on AD-FMM and BSC for MIMO in UWB Systems

Interest in wireless personal area networks (WPANs) has attracted considerable attention (low cost and low power ad-hoc networks are certainly of interest). The ultra-wide band (UWB) technology has emerged as one of the potential candidates for many indoor communication applications such as security systems, wireless home networking, etc. As such, several new standards (i.e. IEEE 802.15.3a and IEEE 802.15.4a) have evolved to investigate the UWB technology for WPAN applications. These new standards imply a need for research in UWB propagation model. Presently (like most propagation channels), UWB channel models are characterized using measurements [62,133]. As such, these channel may only be valid for specific sites. In other word, channel models based on measurements are site-specific in nature, and must be repeated every time for new implementations.
We propose to use the available computational tools (i.e. AD-FMM and BSC) to develop propagation channel models for site-planning. As is well-known, ray tracing (RT) techniques are fast, but not as accurate in modeling penetrable structures with many multipath. As such, detailed penetrable structures within the environment can be modeled by AD-FMM (that is robust, but more computational-intensive). While, the rest can be modeled by RT which is less computational intensive. As a result, the hybridization of AD-FMM and RT is the "best of two worlds" for data collection under many different site configurations and without sacrificing accuracies. Such data can serve to extract fading statistics that can be used to determine performance (e.g. bit error rate (BER)) for different modulation/demodulation techniques (i.e. M-phase shift keying, quadrature amplitude modulation (QAM), orthogonal frequency division multiplexing (OFDM), etc.).

5.2.2 Channel Models for MIMO in 4G Systems

Multimedia applications calls for wireless local area network (LAN) operating over a large bandwidth. In this regard, the 60 GHz band is of great interest because of available spectral space (7 GHz bandwidth) around this frequency. A number of applications, requiring dense wireless local communications, are highlighted in Table 5.1. These indicate that future wireless infra-structure should support real-time traffic. Though other issues, including network flexibility and information integrity play an important role for short-distance dense wireless communications, a good knowledge of propagation channel is still an essential component. Several key considerations are pinpointed in the following.
Table 5.1: Examples of short-range wireless applications [125].

<table>
<thead>
<tr>
<th>Application</th>
<th>Capacity per user [Mb/sec]</th>
<th>Low cost requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wireless LAN bridge</td>
<td>100-1000</td>
<td>No</td>
</tr>
<tr>
<td>Wireless high-quality video-conference</td>
<td>10-100</td>
<td>Yes</td>
</tr>
<tr>
<td>Wireless TV high resolution recording camera</td>
<td>150-270</td>
<td>No</td>
</tr>
<tr>
<td>Wireless interactive design</td>
<td>20-40</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Nevertheless, in spite of the larger bandwidth at 60 GHz, there is much higher propagation loss than at 2.4GHz (IEEE 802.11b) or at 5GHz (IEEE 802.11a). This is because as the frequency increases, the distance in term of wavelength will increase (see Friis’ equation: \( P_{rx} = P_{tx} \frac{G_{tx}G_{rx}\lambda^2}{16\pi^2d^2L} \) where \( P_{rx} \) is the received power, \( P_{tx} \) is the transmitted power, \( G_{tx} \) is transmitter antenna gain, \( G_{rx} \) is receiver antenna gain, \( d \) is the distance between transmitter and receiver in meter, \( \lambda \) is wavelength in meter and \( L \) is system loss). This propagation loss can be overcome by using multiple directional antennas (while concurrently miniaturizing the antenna dimensions). However, deployment of such directional antennas will result in other implementation issues (including obstruction due to nearby objects and misalignment between antennas). These issues have not been addressed rigorously because existing wireless systems are not typically using directive antennas.

Further, at millimeter-wave frequencies, attenuation through wall can be a major factor for reliable connectivity. Specifically, the transmission and reflection coefficients of the wall strongly correlates with the wall material properties and thickness.
at 60 GHz. Due to these correlations, it may be necessary to have a more accurate model for through-wall attenuations.

Another consideration when operating 60 GHz band relates to Doppler effects. Movement of objects at millimeter-wave frequencies can cause severe Doppler shifts since the latter are directly proportional to frequency.

As can be understood, all these considerations are site-specific in nature, implying that work developed in Section 5.2.2 can also be considered in this study.

As a final note, we remark that orthogonal frequency-division multiplex (OFDM) is one potential candidates for transmission of high speed data links at 60GHz. Thus, it is important to consider such transmission scheme in the future work. For OFDM modulation/demodulation, a corrupted frequency-selective wideband channel can still be an effective communication link with the sub-division of a set of narrowband channels (while preserving orthogonality in the frequency domain). In essence, this OFDM scheme is robust against large delay spreads caused by multipath effects. We have been successful in deriving BER for a single carrier phase shift keying (PSK) over a Ricean fading channel (as shown in Appendix B). Thus, BER for OFDM can be considered as an extension of this work (if not made available). The integration of accurate channel model using OFDM will definitely allow for a better understanding of the overall wireless 4G communication system when considered in conjunction with MIMO.
APPENDIX A

STATISTICAL MODELS IN INDOOR PROPAGATION

In order to estimate the channel parameters accurately for wireless systems, it is necessary to compute the propagation characteristics through the medium. The ability to determine these accurately is crucial to system design. Since the measurements are costly, propagation models have been developed as a low-cost and convenient alternative. The existing models can be classified into two major classes: the deterministic and statistical models. These predicts the main characteristics of indoor channel, namely path loss, fading, and time-delay spread. In this appendix, we focus on the statistical models.

A.1 Statistical Models for Small-Scale Fading

Small-scale fading refers to the dramatic changes in the signal amplitude and phase as the result of a small change (i.e. as small as half wavelength) in the spatial separation between a receiver and a transmitter. These dramatic changes in the envelope of the received signal are described statistically by various distributions, namely, Ricean, Rayleigh, Log-Normal, Suzuki, Nakagami and Weibull distributions.
A.1.1 Ricean Distribution

When there is a dominant stationary signal component present, such as line-of-sight propagation path, the fading distribution is Ricean. The Ricean distribution is given by

\[
p(r) = \begin{cases} 
\frac{r}{\sigma^2} \exp \left( -\frac{r^2 + A^2}{2\sigma^2} \right) I_0 \left( \frac{Ar}{\sigma^2} \right) & ; \ A \geq 0, r \geq 0 \\
0 & ; \ r < 0
\end{cases}
\]

where \( r \) is the amplitude of the envelope of the received signal, and \( 2\sigma^2 \) is the predicted mean power of the multipath signal. \( A \) denotes the peak amplitude of the dominant signal, and \( I_0(\bullet) \) is the modified Bessel function of the first kind and zero order.

A.1.2 Rayleigh Distribution

As the dominant signal in a Ricean distribution becomes weaker, the composite signal (noise+signal) has an envelope of a Rayleigh distribution. The Rayleigh distribution has a probability density function (PDF) given by

\[
p(r) = \begin{cases} 
\frac{r}{\sigma^2} \exp \left( -\frac{r^2}{2\sigma^2} \right) & ; \ 0 \leq r \leq \infty \\
0 & ; \ r < 0
\end{cases}
\]

A.1.3 Log-Normal Fading Distribution

This distribution is used to model propagation channel with multiple reflections and diffractions. The log-normal PDF can be expressed as

\[
p(r) = \begin{cases} 
\frac{1}{r\sqrt{2\pi\sigma^2}} \exp \left( -\frac{\ln^2(r) - m^2}{2\sigma^2} \right) & ; \ 0 \leq r \leq \infty \\
0 & ; \ r < 0
\end{cases}
\]

where \( m \) is the median value, and \( \sigma \) is the standard deviation of the corresponding normal distribution.

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A.1.4 Suzuki Model

The Suzuki model combines the log-normal and Rayleigh distribution.

\[
p(r) = \begin{cases} 
  \int_0^\infty \frac{r^2}{\sigma^2} \exp \left(-\frac{r^2}{2\sigma^2}\right) \frac{1}{\sqrt{2\pi}\sigma} \exp \left(-\frac{[\ln(\sigma)-m]^2}{2\sigma^2}\right) d\sigma & ; 0 \leq r \leq \infty \\
  0 & ; r < 0
\end{cases}
\]

where \(\sigma\) is the standard deviation and \(r\) is the amplitude.

A.1.5 Nakagami Model

This model is written as

\[
p(r) = \begin{cases} 
  \frac{2m^{m_r}2m^{-1}\exp\left(-\frac{m_r^2}{\Omega}\right)}{\Gamma(m)\Omega^m} & ; 0 \leq r \leq \infty \\
  0 & ; r < 0
\end{cases}
\]

where \(r\) is the envelope amplitude of the received signal. \(\Omega = \langle r^2 \rangle\) is the time average power of the received signal and \(m = \Omega^2/\text{var}[r^2]\) is the inverse of the normalized variance of \(r^2\). \(\Gamma(\bullet)\) is the Gamma function.

A.1.6 Weibull Model

This model were derived from mobile-radio propagation measurements. The Weibull pdf can be written as

\[
p(r) = \begin{cases} 
  \frac{\alpha b}{r_0} \left(\frac{br}{r_0}\right)^{\alpha-1} \exp \left[-\left(\frac{br}{r_0}\right)^\alpha\right] & ; 0 \leq r \leq \infty \\
  0 & ; r < 0
\end{cases}
\]

where \(\alpha\) is a shape parameter that is chosen to fit the measurement results. \(r_0\) is the RMS value of \(r\) and \(b = [(2/\alpha)\Gamma(2/\alpha)]^{1/2}\) is a normalized factor.
This appendix provides explicit expressions for the BER on pre-defined modulation schemes over various fading channels. Particularly, fading channels of interest are those with Rayleigh, Ricean and Nakagami fading distributions. For simplicity, let us begin with the bit error rate due to the addition white Gaussian noise (AWGN) for a binary pulse amplitude shift keying (ASK) which has two possible signals, namely \( s_1 = A \) and \( s_2 = -A \). In this context, we assume that the probability of \( s_1 = A \) and \( s_2 = -A \) are equal likely. As is well-known, the received signal from a matched filter demodulator (when \( s_1 \) is transmitted) is

\[
r = s_1 + n, \quad (B.1)
\]

where \( n \) represents the AWGN with a normal distribution of zero mean and variance \( \sigma_n^2 = \frac{1}{2} N_0 \). In this derivation, we base on correlation metric with threshold set to zero. That is, \( r > 0 \), the decision is in favor of \( s_1 \) and \( r < 0 \), the decision is in favor of \( s_2 \). As a result, the two probability density functions of \( r \) is
\[
p(r/s_1) = \frac{1}{\sqrt{\pi N_0}} \exp \left[ -\frac{(r - \sqrt{E_b})^2}{N_0} \right]
\]
\[
p(r/s_2) = \frac{1}{\sqrt{\pi N_0}} \exp \left[ -\frac{(r + \sqrt{E_b})^2}{N_0} \right], \quad (B.2)
\]

Given that \(s_1\) was transmitted, the probability of error for \(r < 0\) is

\[
P(e/s_1) = \int_{-\infty}^{0} p(r/s_1) \, dr
\]
\[
= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} \exp \left[ -\frac{(r - \sqrt{E_b})^2}{N_0} \right] \, dr. \quad (B.3)
\]

Performing a change of variable (by letting \(x = \sqrt{\frac{2}{N_0}} \, (r - \sqrt{E_b})\) and \(dr = \sqrt{\frac{N_0}{2}} \, dx\)), it can be shown that (B.3) leads to

\[
P(e/s_1) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} \exp \left[ -\frac{(r - \sqrt{E_b})^2}{N_0} \right] \, dr
\]
\[
= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{-\sqrt{\frac{2E_b}{N_0}}} \exp \left[ -\frac{x^2}{2} \right] \, \sqrt{\frac{N_0}{2}} \, dx
\]
\[
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\sqrt{\frac{2E_b}{N_0}}} \exp \left[ -\frac{x^2}{2} \right] \, dx
\]
\[
= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \exp \left[ -\frac{x^2}{2} \right] \, dx
\]
\[
= Q \left( \sqrt{\frac{2E_b}{N_0}} \right)
\]
\[
= \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right). \quad (B.4)
\]

The similar approach can apply for the case when \(s_2\) is transmitted. That is, \(r = s_2 + n\) and \(P(e/s_1) = P(e/s_2)\). Given that \(s_1\) and \(s_2\) are transmitted with equal probability, the average bit error probability or BER will therefore be
\[
P_b = \frac{1}{2} P(e/s_1) + \frac{1}{2} P(e/s_2) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right).
\]

(B.5)

### B.1 Average Error Rate of binary modulated signal in Rayleigh Fading Channel

As derived earlier, the BER as a function of received SNR \( \gamma_b \) due AWGN channel is

\[
P_2 = \frac{1}{2} \text{erfc}(\sqrt{g\gamma_b}) = Q(\sqrt{2g\gamma_b}),
\]

(B.6)

where \( \gamma_b = \alpha^2 E_b / N_0 \), \( \alpha \) is the attenuation factor (i.e. \( \alpha = 1 \) for non-fading channel), \( g = 1 \) for coherent binary ASK and binary phase shift keying (PSK) and \( g = 0.5 \) for coherent orthogonal binary frequency shift keying (FSK). As we know, \( \alpha \) is random in a fading channel and if we consider the channel to be a Rayleigh fading model, we imply \( \alpha \) is a Rayleigh distributed as shown

\[
P_\alpha = \frac{2\alpha}{\Omega} \exp \left[ -\frac{\alpha^2}{\Omega} \right],
\]

(B.7)

where \( \Omega \) is the shape parameter.

Performing a change of variables (by letting \( \gamma_b = \alpha^2 E_b / N_0 \Rightarrow \alpha = \sqrt{\frac{\gamma_b N_0}{E_b}} \) and \( d\alpha = \frac{1}{2} \left( \frac{\gamma_b N_0}{E_b} \right)^{-\frac{1}{2}} \frac{N_0}{E_b} d\gamma_b \)), we can show that (B.7) becomes

\[
p_{\gamma_b} = \frac{2}{\Omega} \sqrt{\frac{\gamma_b N_0}{E_b}} \exp \left[ -\frac{\gamma_b N_0}{\Omega E_b} \right] \frac{1}{2} \left( \sqrt{\frac{\gamma_b N_0}{E_b}} \right)^{-1} \frac{N_0}{E_b}
\]

\[
= \frac{1}{\gamma_b} \exp \left[ -\frac{\gamma_b}{\gamma_b} \right],
\]

(B.8)
where \( \bar{\gamma}_b = \frac{\Omega E_b}{N_0} \).

Using the identity (7.4.19) of [134], \( \int_0^\infty e^{-at} \text{erf}\sqrt{bt} \, dt = \frac{1}{a} \sqrt{\frac{b}{a+b}} \), the average bit error probability or BER (due to the Rayleigh fading) becomes

\[
P_2 = \int_0^\infty P_2(\gamma_b) p(\gamma_b) \, d\gamma_b
= \int_0^\infty \frac{1}{\gamma_b} e^{-\frac{\alpha^2}{\gamma_b}} Q(\sqrt{2g\gamma_b}) \, d\gamma_b
= \frac{1}{2} \int_0^\infty \frac{1}{\gamma_b} e^{-\frac{\alpha^2}{\gamma_b}} \text{erfc}(\sqrt{g\gamma_b}) \, d\gamma_b
= \frac{1}{2} \int_0^\infty \frac{1}{\gamma_b} e^{-\frac{\alpha^2}{\gamma_b}} [1 - \text{erf}(\sqrt{g\gamma_b})] \, d\gamma_b
= \frac{1}{2} - \frac{1}{2} \left[ \sqrt{\frac{g}{\gamma_b + g}} \right]
= \frac{1}{2} - \frac{1}{2} \left[ \sqrt{\frac{g\gamma_b}{g\gamma_b + 1}} \right].
\] (B.9)

B.2 Average Error Rate of Binary Modulated Signal in Ricean Fading Channel

Next, we look at the BER over the Ricean fading channel. As is well-known, propagation paths that consist of one dominant component (i.e. usually the direct strong line-of-sight (LOS) signal) and many random weak components can be modeled well by Ricean fading channel. That is, the latter channel obeys Ricean distribution as shown

\[
p_\alpha = \frac{\alpha}{\sigma^2} \exp - \left( \frac{\alpha^2 + A^2}{2\sigma^2} \right) I_0 \left( \frac{\alpha A}{\sigma^2} \right),
\] (B.10)

where \( I_0 \) is the modified zero-order Bessel function of the first kind. Performing a change of variables (letting \( \gamma_b = \alpha^2 E_b/N_0 \) and \( d\alpha = \frac{1}{2} \left( \frac{2N_0}{E_b} \right)^{-\frac{1}{2}} N_0 d\gamma_b \)), it can be shown that the SNR per symbol (\( \gamma_b \)) is distributed with association to
\[
p_{\gamma_b} = \frac{\sqrt{\frac{2N_0}{\sigma^2}} \exp \left( -\frac{\gamma_b N_0}{2\sigma^2} + A^2 \right) I_0 \left( \frac{\sqrt{\frac{2N_0}{\sigma^2}} A}{\sigma^2} \right)}{2E_b \sigma^2} \exp \left( -\frac{\frac{\gamma_b N_0}{2\sigma^2} + A^2}{\sigma^2} \right) I_0 \left( \frac{\sqrt{\frac{\gamma_b N_0}{\sigma^2}} A}{\sigma^2} \right)
= \frac{N_0}{2E_b \sigma^2} \exp \left( -\frac{\frac{\gamma_b N_0}{2\sigma^2} + A^2}{\sigma^2} \right) I_0 \left( \frac{\sqrt{\frac{\gamma_b N_0}{\sigma^2}} A}{\sigma^2} \right)
= \frac{1}{\gamma_b} \exp \left[ -\left( \frac{\gamma_b}{\gamma_b} + K \right) \right] I_0 \left( \frac{4K \gamma_b}{\gamma_b} \right), \quad (B.11)
\]

where \( K = \frac{A^2}{2\sigma^2} \) and \( \gamma_b = \frac{2\sigma^2 E_b}{N_0} \).

Therefore, the average BER due to Ricean fading is as follow

\[
P_2 = \int_0^\infty P_2(\gamma_b) p(\gamma_b) \, d\gamma_b
= \int_0^\infty \frac{1}{\gamma_b} \exp \left[ -\left( \frac{\gamma_b}{\gamma_b} + K \right) \right] I_0 \left( \frac{4K \gamma_b}{\gamma_b} \right)
\cdot Q(\sqrt{2g \gamma_b}) \, d\gamma_b
= \frac{1}{\gamma_b \pi} \int_0^{\pi/2} \int_0^\infty \exp \left[ -\left( \frac{\gamma_b}{\gamma_b} + K \right) \right] I_0 \left( \frac{4K \gamma_b}{\gamma_b} \right)
\cdot \exp \left( -\frac{g \gamma_b}{\sin^2 \phi} \right) \, d\gamma_b \, d\phi
= \frac{1}{\gamma_b \pi} \int_0^{\pi/2} \int_0^\infty \exp \left[ -\left( \frac{\gamma_b}{\gamma_b} + K \right) \right] I_0 \left( \frac{4K \gamma_b}{\gamma_b} \right)
\cdot \exp \left( -\frac{g \gamma_b}{\sin^2 \phi} \right) \, d\gamma_b \, d\phi
= \frac{1}{\gamma_b \pi} \exp(-K) \int_0^{\pi/2} \int_0^\infty \exp \left[ -\gamma_b \left( \frac{\sin^2 \phi + g \gamma_b}{\gamma_b \sin^2 \phi} \right) \right] I_0 \left( \frac{4K \gamma_b}{\gamma_b} \right) \, d\gamma_b \, d\phi.
\quad (B.12)
\]

Using the Laplace transform [134, (29.3.81)], \( \int_0^\infty I_0(u\sqrt{x}) \exp(-sx) \, dx = \frac{\exp(u^2/(4s))}{s} \), \( s > 0 \), (B.12) can be simplified as shown
\[ P_2 = \frac{1}{\gamma_b \pi} \exp(-K) \int_0^{\pi/2} \exp \left[ \frac{4K}{\gamma_b} \left( \frac{ \sin^2 \phi + g \gamma_b}{\sin^2 \phi} \right) \right] \left( \frac{ \sin^2 \phi + g \gamma_b}{\sin^2 \phi} \right) d\phi \]
\[ = \frac{1}{\pi} \exp(-K) \int_0^{\pi/2} \exp \left[ \frac{K \sin^2 \phi}{\sin^2 \phi + 2g \gamma_b} \right] \left( \frac{ \sin^2 \phi}{\sin^2 \phi + g \gamma_b} \right) d\phi \]
\[ = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{ \sin^2 \phi}{\sin^2 \phi + g \gamma_b} \right) \exp \left[ -K g \gamma_b \right] d\phi. \quad (B.13) \]

\[ \text{B.3 Average Error Rate of Binary Modulated signal in Nakagami-n Fading Channel} \]

Ricean-behavior like fading channel (i.e. propagation paths that one dominant component and many weak multipath components) can also be modeled by the Nakagami-n distribution. The Nakagami-n distribution is given in [83, (50)] as

\[ p(R) = \frac{2R}{\sigma} \exp \left[ -(R^2 + R_0^2)/\sigma \right] I_0 \left( \frac{2RR_0}{\sigma} \right) \equiv \mathcal{N}(R,R_0,\sigma). \quad (B.14) \]

Performing a change of variable (by letting \( R_0 = n \sqrt{\frac{\Omega}{1+n^2}} \), \( \frac{1}{\sigma} = \frac{1+n^2}{\Omega} \), and \( R = \alpha \)), (B.14) becomes

\[ p_a = \frac{2\alpha(1+n^2)}{\Omega} \exp \left[ -\left( (1+n^2)\alpha^2 + n^2 \right) \right] I_0 \left( 2\alpha n \sqrt{\frac{1+n^2}{\Omega}} \right), \quad (B.15) \]

where \( n \) is the Nakagami-n fading parameter that ranges from 0 to \( \infty \) and is, in fact, related to Ricean \( K \)-factor by \( K = n^2 \). Again, performing a change of variables, it can be shown that the SNR per symbol \( \gamma_b \) is distributed with association to
\[ p_{\gamma_b} = \frac{2(1 + n^2)}{\Omega} \sqrt{\frac{\gamma_b N_0}{E_b}} \exp \left[ - \left( \frac{(1 + n^2)^2 \gamma_b N_0}{\Omega E_b} \right) + n^2 \right] I_0 \left[ 2n \sqrt{\frac{\gamma_b N_0 \Omega}{E_b}} \right] \]

\[ \frac{1}{2} \left( \frac{\gamma_b N_0}{E} \right)^{-\frac{1}{2}} \left( \frac{N_0}{E_b} \right) \]

\[ = \frac{(1 + n^2)}{\gamma_b} \exp \left[ - \left( \frac{(1 + n^2) \gamma_b}{\gamma_b} \right) + n^2 \right] I_0 \left[ 2n \sqrt{\frac{(1 + n^2) \gamma_b}{\gamma_b}} \right]. \tag{B.16} \]

Therefore, the average BER due to Nakagami-n fading is as follow

\[
P_2 = \int_0^{\infty} P_2(\gamma_b)p(\gamma_b) \, d\gamma_b
\]

\[
= \int_0^{\infty} \frac{(1 + n^2)}{\gamma_b} \exp \left[ - \left( \frac{(1 + n^2) \gamma_b}{\gamma_b} \right) + n^2 \right] I_0 \left[ 2n \sqrt{\frac{(1 + n^2) \gamma_b}{\gamma_b}} \right] \cdot Q(\sqrt{2g\gamma_b}) \, d\gamma_b. \tag{B.17}
\]

Further simplification is possible by using the Laplace transform \[134, (29.3.81)],

\[
\int_0^{\infty} I_0(u\sqrt{x}) \exp(-sx) \, dx = \frac{\exp(u^2/(4s))}{s}, (s > 0). \]

As such, (B.17) leads to
\[ P_2 = \frac{1}{\pi} \int_0^{\pi/2} \int_0^{\infty} \frac{(1 + n^2)}{\tilde{\gamma}_b} \exp(-n^2) \exp \left[ -\frac{(1 + n^2)\gamma_b}{\tilde{\gamma}_b} \right] \\
\cdot I_0 \left[ 2n\sqrt{\frac{(1 + n^2)\gamma_b}{\tilde{\gamma}_b}} \exp \left[ -\frac{g\gamma_b}{\sin^2 \phi} \right] \right] d\gamma_b d\phi \\
= \frac{1}{\pi} \frac{(1 + n^2)}{\tilde{\gamma}_b} \exp(-n^2) \int_0^{\pi/2} \int_0^{\infty} \exp \left[ -\gamma_b \left( \frac{(1 + n^2)\gamma_b}{\tilde{\gamma}_b} + \frac{g}{\sin^2 \phi} \right) \right] \\
\cdot I_0 \left[ 2n\sqrt{\frac{(1 + n^2)\gamma_b}{\tilde{\gamma}_b}} \right] d\gamma_b d\phi \\
= \frac{1}{\pi} \frac{(1 + n^2)}{\tilde{\gamma}_b} \exp(-n^2) \int_0^{\pi/2} \exp \left[ 4n^2 \frac{(1 + n^2)}{\tilde{\gamma}_b} \right] / \left[ 4 \left( \frac{(1 + n^2)}{\tilde{\gamma}_b} + \frac{g}{\sin^2 \phi} \right) \right] \\
\cdot \left( \frac{(1 + n^2)}{\tilde{\gamma}_b + \frac{g}{\sin^2 \phi}} \right) d\phi \\
= \frac{1}{\pi} \frac{(1 + n^2)}{\tilde{\gamma}_b} \exp(-n^2) \int_0^{\pi/2} \exp \left[ n^2 \frac{(1 + n^2)}{\tilde{\gamma}_b} \right] \left[ \frac{(1 + n^2)\sin^2 \phi + g\tilde{\gamma}_b}{\gamma_b \sin^2 \phi + g}\right] \\
\cdot \left( \frac{\tilde{\gamma}_b \sin^2 \phi + g\tilde{\gamma}_b}{(1 + n^2) \sin^2 \phi + g\tilde{\gamma}_b} \right) d\phi \\
= \frac{1}{\pi} \int_0^{\pi/2} \frac{(1 + n^2) \sin^2 \phi}{(1 + n^2) \sin^2 \phi + g\tilde{\gamma}_b} \exp \left[ -\frac{n^2 g\tilde{\gamma}_b}{(1 + n^2) \sin^2 \phi + g\tilde{\gamma}_b} \right] d\phi. \quad (B.18) \]

### B.4 Average Error Rate of Binary Modulated Signal in Nakagami-m Fading Channel

Last, but the least, another important fading model that we introduce in this appendix is Nakagami-m. The latter distribution is given in [83, (11)] and [82, (8)] as

\[ p_\alpha = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} \exp \left( -\frac{m\alpha^2}{\Omega} \right). \quad (B.19) \]
Performing a change of variables, it can be shown that the SNR per symbol ($\gamma_b$) is distributed with association to

$$p_{\gamma_b} = \frac{2m^m (\alpha^2)^m \alpha^{-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m \alpha^2}{\Omega}\right)$$

$$= \frac{2m^m \left(\frac{\gamma_b N_0}{E_b}\right)^m \left(\frac{\gamma_b N_0}{E_b}\right)^{-\frac{1}{2}}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m \gamma_b N_0}{\Omega E_b}\right) \frac{1}{2} \left(\frac{\gamma_b N_0}{E_b}\right)^{-\frac{1}{2}} \frac{N_0}{E_b}$$

$$= \frac{m^m \left(\frac{\gamma_b N_0}{E_b}\right)^m \left(\frac{\gamma_b N_0}{E_b}\right)^{-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m \gamma_b N_0}{\Omega E_b}\right) \frac{N_0}{E_b}$$

$$= \frac{m^m \left(\frac{\gamma_b N_0}{E_b}\right)^m \left(\gamma_b\right)^{-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m \gamma_b N_0}{\Omega E_b}\right)$$

$$= \frac{m^m \gamma_b^{m-1}}{\gamma_b^m \Gamma(m)} \exp\left(-\frac{m \gamma_b}{\gamma_b}\right).$$

(B.20)

Therefore, the average BER due to Nakagami-m fading is

$$P_2 = \int_0^\infty P_2(\gamma_b) p(\gamma_b) d\gamma_b$$

$$= \int_0^\infty \frac{m^m \gamma_b^{m-1}}{\gamma_b^m \Gamma(m)} \exp\left(-\frac{m \gamma_b}{\gamma_b}\right) Q\left(\sqrt{2g \gamma_b}\right) d\gamma_b. \quad \text{(B.21)}$$

Further simplification using the Laplace transform $[134, (29.3.7)]$, $\int_0^\infty \frac{x^{v-1}}{s} \exp(-sx) dx = \frac{\Gamma(v)}{s^v}, (s > 0)$ will result (B.21) to
\[ P_2 = \frac{1}{\pi} \int_0^{\pi/2} \int_0^\infty \frac{m^m \gamma_b^{-m-1}}{\Gamma(m)} \exp \left( -\frac{m \gamma_b}{\gamma_b} \right) \exp \left[ -\frac{g \gamma_b}{\sin^2 \phi} \right] d\gamma_b d\phi \]

\[ = \frac{1}{\pi} \int_0^{\pi/2} \int_0^\infty \frac{m^m \gamma_b^{-m-1}}{\Gamma(m)} \exp \left[ -\gamma_b \left( \frac{m}{\gamma_b} + \frac{g}{\sin^2 \phi} \right) \right] d\gamma_b d\phi \]

\[ = \frac{1}{\pi} \int_0^{\pi/2} m^m \gamma_b^{-m-1} \Gamma(m) \left( \frac{m}{\gamma_b} + \frac{g}{\sin^2 \phi} \right)^m d\phi \]

\[ = \frac{1}{\pi} \int_0^{\pi/2} \gamma_b^{-m} \left( m \sin^2 \phi + g \gamma_b \right)^m d\phi \]

\[ = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\gamma_b^{-m}}{m \sin^2 \phi} \right)^m d\phi. \]  

(B.22)
APPENDIX C

GOODNESS-OF-FIT

This section demonstrate various goodness-of-fit techniques available for testing the validity of the fit.

C.1 Pearson’s Chi-Square Goodness-of-Fit

The chi-square goodness-of-fit is used to analyze binned data. There is no restricted to non-binned data, as the latter can be easily categorized into different bins (i.e. histogram), before applying chi-square test. However, the accuracy of the test depend on how the data is binned. Another disadvantage of the chi-square test is that it requires a sufficient sample size to be valid.

C.1.1 Definition

\( H_0 : \) The data follow a specific distribution

\( H_A : \) The data does not follow a specific distribution

Test Statistics : \( T = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \)

where \( O_i \) is the observed frequency for bin \( i \), \( E_i \) is the expected frequency for bin \( i \), and \( k \) is the total number of bins.
C.1.2 Test

We do not accept the $H_0$ hypothesis if

\[ T > \chi^2_{(\alpha, N)}, \]  

(C.1)

where $N = (\text{number of bins} - \text{number of independent parameters fitted} - 1)$ and $\chi^2_{\alpha, N}$ is the chi-square distribution with $N$ degree of freedom and a significant level of $\alpha$.

C.2 Kolmogorov-Smirnov Goodness-of-Fit

The Kolmogorov-Smirnov (KS) test is also an alternative test to decide whether an observed distribution comes from a specific distribution. An attractive feature of KS test is that its statistics does not depend on the expected distribution being tested. Another advantage is that the KS test is exact, unlike the chi-square test which depends on the sample size. In spite of these advantages, KS test has its share of drawbacks, namely (1) it applies to continuous distribution, (2) it tends to be more sensitive near the center of the distribution than at the tails, and (3) it is difficult to determine the critical region when location, shape and shape parameters are estimated from the data.

C.2.1 Definition

\[ H_0 : \text{ The data follow a specific distribution} \]

\[ H_A : \text{ The data does not follow a specific distribution} \]

Test Statistics : \[ D = \sqrt{N} \max_{1 \leq i \leq N} \left| F(Y_i) - \frac{i}{N} \right| \]
where $N$ is total number of data points, $F$ is the theoretical cumulative distribution function which is continuous and must be fully specified, and $Y_i$ are the ordered from the smallest to the largest value of the empirical cumulative distribution function.

C.2.2 Test

We do not accept the $H_0$ hypothesis if

$$D > 1.36 \quad \text{for } \alpha = 0.05$$

$$D > 1.67 \quad \text{for } \alpha = 0.01.$$ 

C.3 Anderson-Darling Goodness-of-Fit

The Anderson-Darling (AD) test like the two aforementioned tests is used to determine whether a sample of data came from a specific distribution. It is a modification of KS test where AD test is more sensitive on the tails than the center of the distribution. The disadvantage of the AD test is that critical value must be calculated for each specific distribution tested. To the author’s best knowledge, tables of critical values are available for the normal, log-normal, exponential, Weibull and extreme value type I and logistic distributions. Since critical value is so specific, AD test is a more sensitive test.

C.3.1 Definition

$H_0$ : The data follow a specific distribution

$H_A$ : The data does not follow a specific distribution

Test Statistics : $A^2 = -N - \sum_{i=1}^{N} \frac{(2i-1)}{N} \left[ \log F(Y_i) + \log(1 - F(Y_{N+1-i})) \right]$
where $N$ is total number of data points, $F$ is the theoretical cumulative distribution function and $Y_i$ are the ordered from the smallest to the largest value of the empirical cumulative distribution function.

**C.3.2 Test**

We do not accept the $H_0$ hypothesis if

$$A > CV.$$  \hspace{1cm} (C.2)

$CV$ is the critical value and there is a number of papers that contain tables of critical values for different specific distributions.
BIBLIOGRAPHY


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