Ratio relationships between π, the fine structure constant and the frequency equivalents of an electron, the Bohr radius, the ionization energy of hydrogen, and the classical electron radius

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Abstract:

This paper demonstrates and analyzes the multiple complicated ratio inter-relationships of pi, π, the fine structure constant, α, and the frequency equivalents of an electron, v_e, the Bohr radius, v_{a0}, the ionization energy of hydrogen or the Rydberg constant, R, v_R, and the classical electron radius, v_{re}. These relationships are in part known and many unknown. This paper focuses on all of them as a unified system. They are important since they demonstrate an unexpected inter-relationship of quantum properties of hydrogen that can be used to calculate and understand relationships between fundamental constants. These ratios are of logical origin, but quite complicated and not intuitive. These relationships are related to geometry and the nature of the components of hydrogen, for example a transition of a mass/ energy to a distance. α is related to four different frequency equivalent ratios including: the product of 4π and v_R, divided by v_{a0}, the square root of 2 times v_r divided by v_e, v_{a0} divided by the product of 2π times v_e, and the product of 2π and v_e divided by v_{re}. π is related to three ratios of α and the hydrogen frequency equivalents from the previous relationships. $8\pi^2$ is equal to the ratio of v_{a0} squared divided by the product of v_r and v_e. The combination products of α, 2 and 2π are the spacing ratios between the transformation between a mass/energy and a distance for all of the four hydrogen constants. The ratio of v_R to v_{re} related to α^3 divided by 4π. If any three of the six constants are known then the other three can be derived demonstrating how intricately related these constants are. These relationships also must hold for the effective α as well, and in part explain its origin and why α must change in certain settings. Therefore these relationships extend to high energy physics as well as adding insights to effective α.

Key words: fine structure constant, electron, Bohr radius, Rydberg constant, fundamental constants, pi

Introduction:

It is common to encounter inter-relationships between fundamental constants and π. This paper explores the ratio inter- relationships of the quantum properties of hydrogen. Since the fine structure constant, α, and pi, π are both dimensionless number ratios all these properties of hydrogen are converted to frequency equivalents and evaluated as dimensionless ratios [1, 2]. The entities are evaluated as frequency equivalents and include: an electron, v_e, the Bohr radius, v_{a0}, and the ionization energy of hydrogen or the Rydberg constant, R, v_R, and the classical electron radius, v_{re}.

It will be shown that these frequency equivalents related to the components defining hydrogen are directly related to π and α. These in part represent classic Euclidean geometric relationships, but they are within a quantum system across physical constant units of energy/ mass, and distance. Though complicated these findings are anticipated, but they are not intuitive. Utilizing one unit, Hz, makes these relationships apparent. These relationships are important since they demonstrate the inherent fundamental inter-connections between the properties of hydrogen. These relationships must also be fulfilled for the effective α, running α, so these relationships extend beyond hydrogen into high energy physics as well. These finding illustrate insights into effective α.

Methods and Results:

All of the fundamental constants are converted to frequency equivalents [1, 2]. The masses are converted by multiplying by c^2 (speed of light squared) then dividing by h (Planck constant). The distances are converted by dividing the wavelength into c. Energies are converted by dividing by h. v_e equals 1.2355899 x 10^{20} Hz, v_{a0} equals 5.6652564 x 10^{18} Hz. v_R equals 3.28984196 x 10^{15} Hz. Classical electron radius, v_{re} equals 1.06387087 x 10^{23} Hz.

All of the data for the fundamental constants is from website: http://physics.nist.gov/cuu/Constants/

Equation (1) is the classic Coulomb's law where F equals the force, k, Coulomb's constant, e^2, the square of charge of an electron, divided by the distance separating the charges squared, λ^2. In this paper the binding energy of the electron in hydrogen is used as the unified quantum system. Equation (2) evaluates the energy of a system rather than the force. A different representation of Coulomb's constant, k, is utilized with components of the speed of light squared, c^2, and the magnetic constant, μ_0. In equation (3) both sides of equation (2) are divided by Planck's constant, h which converts the energies into frequency equivalents. In this case the frequency equivalent of the electron binding energy of hydrogen v_R is shown. This is related to the Rydberg constant, R. Rearranging the other components converts the Bohr radius, λ_{a0}, to v_{a0} seen in equation (4).

$$F = \frac{ke^2}{\lambda^2} \quad (1)$$

$$E_R = \frac{c^2\mu_0}{4\pi}\frac{e^2}{2\lambda_{a0}} \quad (2)$$

$$\frac{E_R}{h} = \frac{c\mu_0 e^2}{h}\frac{1}{8\pi}\frac{c}{\lambda_{a0}} \quad (3)$$

$$v_R = \frac{c\mu_0 e^2}{h}\frac{1}{8\pi}v_{a0} \quad (4)$$

Equation (5) demonstrates that the dimensionless product of c, μ_0, e^2, divided by h. This is known to be equal 2α.

Substituting this back into equation (4) leads to equation (6) where α is related to 4 π times v_R divided by v_{a0}. Equation (7) solves for π.

$$2\alpha = \frac{c\mu_0 e^2}{h} \quad (5)$$

$$\alpha = \frac{4\pi v_R}{v_{a0}} \quad (6)$$

$$\pi = \frac{v_{a0}\alpha}{4v_R} \quad (7)$$

It is known that the ionization energy of hydrogen conceptually is related to the change in velocity of an electron from the annihilation velocity of c to αc, Equation (8). Dividing the energies on both sides of the equation by h converts this relationship to frequency equivalents, Equations (9). Equations (10, 11) solves for alpha. There is no π in this equation. α^2 divided by 2 equals the ratio of v_R divided by v_e. Rearranging equation (10) solves for v_R, equation (12). Substituting this into equation (7) demonstrates that α equals v_{a0} divided by the product of 2π and v_e, Equation (13). π is solved for in Equation (14).

$$E_R = \frac{m_e c^2 \alpha^2}{2} \quad (8)$$

$$\frac{E_R}{h} = v_R = \frac{m_e c^2}{h}\frac{\alpha^2}{2} = v_e \frac{\alpha^2}{2} \quad (9)$$

$$\frac{\alpha^2}{2} = \frac{v_R}{v_e} = \frac{v_{a0}}{v_e}\frac{v_R}{v_{a0}} \quad (10)$$

$$\alpha = \sqrt{\frac{2v_R}{v_e}} \quad (11)$$

$$v_R = \frac{v_e \alpha^2}{2} \quad (12)$$

$$\alpha = \frac{v_{a0}}{2\pi v_e} \quad (13)$$

$$\pi = \frac{v_{a0}}{2\alpha v_e} \quad (14)$$

The ratio of v_{a0} divided by v_e, divided by the ratio of v_R divided by v_{a0} cancels out all of the α factors leaving only $8\pi^2$, Equation (15). This is equal to v_{a0} squared divided by the product of v_e and v_R. Equation (15). Equation (16) solves for π solely from quantum constants of hydrogen.

$$8\pi^2 = \frac{{v_{a0}}^2}{v_e v_R} = \frac{\left(\frac{v_{a0}}{v_e}\right)}{\left(\frac{v_R}{v_{a0}}\right)} \quad (15)$$

$$\pi = \frac{v_{a0}}{2\sqrt{2v_e v_R}} \quad (16)$$

r_e is related to the distance separating two unit charges with a potential annihilation energy equal to the mass of an electron, Equation (17). This is in a similar form as equation (2). Equation (18) divides both sides by h converting the values to frequency equivalents. In Equation (19) α equals the product of 2π times v_e divided by v_{re}. Equation (20) rearranges the components. Equation (21) solves for π.

$$E_e = \frac{c^2 \mu_0}{4\pi} \frac{e^2}{r_e} \quad (17)$$

$$\frac{E_e}{h} = \frac{c\mu_0 e^2}{h} \frac{1}{4\pi} \frac{c}{r_e} = v_e = \frac{\alpha v_{re}}{2\pi} \quad (18)$$

$$\alpha = \frac{2\pi v_e}{v_{re}} \quad (19)$$

$$v_{re} = \frac{2\pi v_e}{\alpha} \quad (20)$$

$$\pi = \frac{\alpha v_{re}}{2 v_e} \quad (21)$$

Equations (22) are known relationships of r_e associated with a_0 and the Compton radius of the electron, λ_e. Dividing both sides by c converts them to frequency equivalents. The relationships are related to the square of α are shown in Equation (23). There is no 2 in the relationship with v_{re} as in Equation (24) since this represents a potential energy.

$$r_e = \frac{\alpha \lambda_e}{2\pi} = \alpha^2 a_0 \quad (22)$$

$$\frac{r_e}{c} = \alpha^2 \frac{a_0}{c} = \frac{1}{v_{re}} = \frac{\alpha^2}{v_{a0}} \quad (23)$$

$$\alpha^2 = \frac{v_{a0}}{v_{re}} = \frac{2 v_R}{v_e} \quad (24)$$

Equation (25) demonstrates the relationship of ratio of the smallest and largest hydrogen components from v_R to v_{re}. In this case it is related to α^3 divided by 4π.

$$\frac{\alpha^3}{4\pi} = \frac{v_R}{v_{re}} \quad (25)$$

Figure 1 plots these relationships as log values. This demonstrates these relationships more clearly. Many of the possible sums and differences of the log values of 2, 2π, and α exist as ratios of the actual physical components of hydrogen. Each transition from one entity to the next is a combination related to factors of α, 2 or 2π.

Discussion:

π is a ubiquitous constant in physics. It is frequently incorporated into the definition of constants. In this specific system these relationships are logical, but not intuitive since they are so convoluted. Alpha can be conceptually viewed as the change in the velocity of the electron from annihilation speed of c to αc, Equation (8). Therefore the ratio of v_R and v_e are related to α^2, Equation (10). This energy is then related to the ionization energy of hydrogen. The factor 2 is related to that fact that this is not a potential energy which is related to the definition of α as a potential energy. The ionization energy is similar to a kinetic energy versus an annihilation energy. The 2π factor is related to the transformation of a radius to a circumference which effectively elongates the wavelength equivalent. This is related to the ratio between the a_0 and a circular orbit of the electron, Equation (13). These two factors account for 4π of $8\pi^2$in Equation (15).

The second 2π factor associated with the $8\pi^2$ ratio arises from the fact that the total ratio relationship of v_R and v_e is defined by Equation (10). Figure 1 demonstrates that any added factor related to the ratio of v_R and v_{a0} must be cancelled out by the ratio relationship of v_{a0} and v_e. The ratio of these two ratios, Equation (15), adds the second 2π. The α factors cancel.

Equation (16) equals π from solely the frequency equivalents of quantum properties of hydrogen and integers only. This does not describe the actual geometry of hydrogen in a classic Euclid geometric sense, but is the result of dividing the ratios between v_{a0} and v_e by v_R and v_{a0}, Figure 1. This is a specific situation where three fundamental quantum constant values of identical units are all related to just integers and π.

r_e is also defined by these same hydrogen factors, therefore also related to integers, α, and π. This is logical since it is very similar to the relationship between v_R and v_{a0} in Equations (2, 17). It is related to a potential energy though. That accounts for the difference from the relationship of v_R and v_{a0}.

Figure 1 demonstrates the relative values of these ratios in the log format. It is clear that many of the different possible sums of log α, log 2, log ½, log 2π, and log $1/2\pi$ are associated with these four fundamental constants and there inter-relationships. α^2 is related to the spread from v_R to v_e, or v_{a0} to v_{re}. α^3 is related to the maximum ratio transition from v_R to v_{re}. This series alternates between constants related to mass/energy and distance constants. Each transition is scaled by a factor of α in combination with 2 or 2π. A transition from and mass/ energy to mass/ energy or distance to distance is not associated with π, but α^2. This is true since there is not transition from a radius to a circumference.

This is a very complicated, but exact "dance" between all six of these constants. It is possible to derive all six if only three of the physical constants are known. It is not typical to consider calculating α from many of these properties, but this is valid.

These ratio inter-relationships between all of these factors must also be fulfilled for effective α, running α, as well. This in part explains why there is an effective α at all. The only mathematical means to maintain all of these four ratio relationships, Equations (6, 11, 13, 15) is for α to change. The Bohr radius can be viewed as the "quantum equilibrium" separation of two changes with a specific energy. This is the "natural" distance that is a quantum minimum. For a distance separation of unit charges less than the Bohr radius in hydrogen is not "stable" and is associated with a change in α. In this setting the mass of the electron can be viewed as a constant. The Bohr radius would be less, therefore would have a higher value for the frequency equivalent than v_{a0}. The other factor that must change is the frequency equivalent of the energy. If the inter-relationships were related to the simple ratio values of any two factors alone then running α would not have to exist. In Equation (15) though v_{a0} is squared. This "forces" the α value to change to maintain this relationship.

? do calculation of change in vao and see what happens.

Equation (15) is significant since it is related to properties of mass, distance, and energy all within hydrogen. The factor π demonstrates that there is a deep fundamental geometric relationship that is unified across these different physical entities and properties. Evaluating them in a common unit allows for this observation that would otherwise be obscured by the different units.

Figures:

Figure 1.

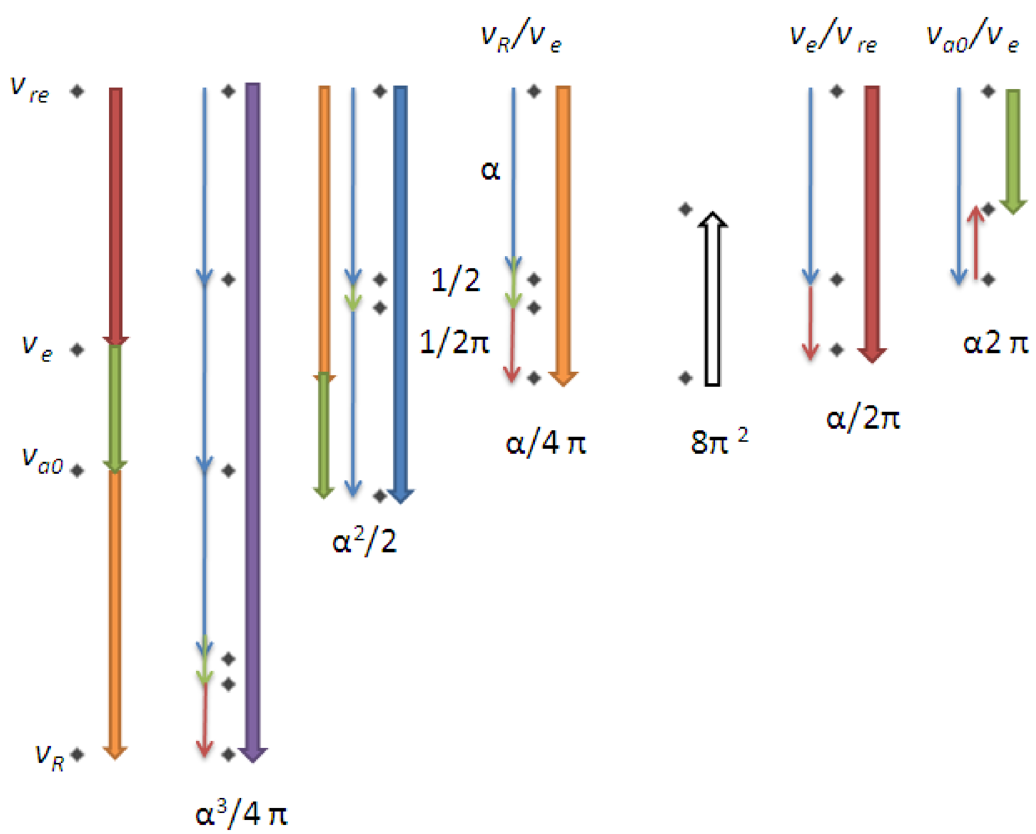

Figure 1. This figure plots the log values for α, 2π and 2. Log α is the thin blue arrow. Log 2π is the thin red arrow. Log 2 is the thin green arrow. The associated log ratio values of the fundamental constants related to the frequency equivalents of v_{re}, v_e, v_{a0}, and v_R are also shown and their inter- relationships. The

thick green arrow is ratio v_{a0}/v_e. The thick red arrow is the ratio v_e/v_{re}. The thick orange arrow is v_R/v_e. Most of the possible combinations of these factors exist as actual physical entities. Note that the series alternates from a distance to a mass/energy entity. There is a ratio change of α scaled by either 2, 2π, both or neither. The only value that is independent of α is the $8\pi^2$ ratio, the open black arrow. The ratio of v_R/v_e is the thick blue arrow and is related to α^2. The purple arrow is related to v_R/v_{re} and is related to $\alpha^3/4\pi$.

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